THE
MATHEMATICAL GAZETTE

EDITED BY
T. A. A. BROADBENT, M.A.
ROYAL NAVAL COLLEGE, GREENWICH, LONDON, S.E. 10.

LONDON
G. BELL AND SONS, LTD., PORTUGAL STREET, KINGSWAY

VOL. XXIX. OCTOBER, 1945. No. 286

EDDINGTON'S THEORY OF THE CONSTANTS OF NATURE.

BY SIR EDMUND WHITTAKER.

SIR ARTHUR EDDINGTON died on 22nd November, 1944. The last sixteen years of his life had been occupied chiefly with the discovery and development of certain new principles in physics, which he used in the first place in order to calculate theoretically the " Constants of Nature " (such as the ratio of the mass of the proton to the mass of the electron); he did in fact succeed in deriving correctly the values of all those constants of Nature which are pure numbers. Later he showed that his ideas could be presented as a coherent system, to which he gave the name *Fundamental Theory,* and which may be broadly summed up in the statement: *the external world is isomorphous with the square of quaternion algebra.*

The earlier papers had the defects of form and logic which are almost inevitable in pioneer work, and were severely criticised on that account. The prejudice thus generated, together with the mathematical difficulties of non-commutative algebra, have prevented any wide knowledge or general appreciation of the extraordinary power and significance of the theory. There has indeed been considerable misunderstanding about the character of the entire series of investigations; it has sometimes been supposed (erroneously) that Eddington claimed to have derived the whole of physics, or a large part of it, by pure cogitation, without depending in any way on the results of observation and experiment. Such an idea is refuted at once by an examination of the papers themselves. The position may be illustrated by reference to the history of an older problem. The ancient Egyptians were acquainted with the fact that the ratio of the area of a circle to the square on its radius was independent of the size of the circle; and for this number, which we denote by π, they found, by actual measurement, the value $\frac{256}{81}$ ($=3{\cdot}16\ldots$). In the third century before Christ, Archimedes showed that the number can be found to any desired degree of accuracy by pure theory, without the necessity for making measurements. For this purpose he assumed the axioms and propositions of geometry as they had been set forth in the preceding generation by Euclid; so that what Archimedes did was to assume the *qualitative* part of geometry and to deduce a *quantitative* aspect of it, namely the number π.

Now Eddington is simply the modern Archimedes. He regarded himself as at liberty to borrow anything in *qualitative* physics—he did in fact assume the

identity of mass and energy, the theory of the energy-tensor and the interpretation of its elements, the exclusion principle, and other propositions of the most advanced physical theory—but he did *not* assume any number determined empirically; and he deduced the quantitative propositions of physics, *i.e.* the exact values of the pure numbers that are constants of science—the numbers that are analogous to the number π in geometry.

One can imagine the protests of those contemporaries of Archimedes who had been accustomed to find π by measuring circles, when he took the bread out of their mouths by his mathematical evaluation; and a similar outcry was raised by certain experimental physicists against Eddington, who, it must be admitted, sometimes played into their hands by using language that was liable to be misunderstood. For instance, he obtained for "the number of particles in the universe" a value which, his critics complained, could never be checked by observation; actually, as we shall see, the number thus referred to was carefully defined, and has a value which can be determined independently of Eddington's work by a combination of orthodox relativity-theory and astronomical measurements.

The investigation of the constants of Nature may be regarded as, in some sense, a continuation of certain researches in relativity-theory on which Eddington had been engaged for many years previously. But the impetus which led to the new development came with the publication, in February 1928, of a celebrated paper by Dirac on the wave-equation of the electron, in which it was shown that the power of the quantum theory to explain atomic spectra could be greatly increased by introducing relativistic ideas. Both quantum mechanics and relativity were now well-developed mathematical theories, each remarkably successful in its own domain; and as they represented complementary aspects of the world (relativity finding most of its applications in the astronomical field, while quantum mechanics was studied chiefly in connection with atomic physics), Eddington felt that the time had come to construct a comprehensive doctrine combining and transcending both. He realised that while each theory separately had attained remarkable power as regards predicting the future—foretelling what would happen in the course of a prolonged observation—it would be necessary to look to some combination of the two in order to determine the ultimate *structure* of things, *e.g.* to deduce from theory the ratio of the masses of a proton and an electron.

The first stage was to find a common meeting-point of relativity and quantum-theory—to consider a problem which could be solved rigorously by both methods. Such a problem is the state of equilibrum of a radiationless self-contained system of a very large number of particles. In molar relativity this problem was first solved by Einstein. The presence of the matter produces a curvature of space, depending on the total number of particles, say N; so we obtain a closed curved space, say of radius R, in which the mutual gravitational attraction of the particles is exactly balanced by their mutual repulsion due to the cosmical term in General Relativity. Such a system is called an *Einstein universe.*

Passing now to the other aspect, in quantum mechanics a radiationless steady system is said to be in its *ground state.* We have therefore to determine the conditions to be satisfied by a system of N particles treated (a) as an Einstein universe with zero pressure and temperature, (b) as a quantised system in its ground state. The two answers must agree; and since the relativity solution will be expressed in terms of the gravitational constant κ and the cosmical constant λ, whereas the quantum solution will be expressed in terms of Planck's constant h and other microscopic constants, it is evident that a comparison of the two solutions will yield a relation between the constants of Nature.

In the quantum solution energy is represented in terms of wave functions, so the distribution is analysed into orthogonal wave functions (the surface harmonics of the hypersphere of space) which correspond to possible energy-levels of the particles. We assume the "exclusion principle" which forbids the multiple occupation of any one energy-level,* and thus we conclude that in the ground state, the N particles will occupy the N states of lowest energy, represented by the surface harmonics of lowest order. The total energy of the Einstein universe is thus found in terms of N and R and the quantum constants; and the result is to be equated to the energy (or mass) of an Einstein universe as given by relativity-theory. The value of N/R is known from relativity-theory, and thus we have two equations which enable us to find N and R separately.

Eddington's original method (1931) of finding N theoretically was heuristic in character and has been superseded by his later work, but it is of interest as revealing the psychological process of discovery. The idea was that in the ordinary Dirac wave equation for an electron moving in the electrostatic field due to a fixed electric charge, the term which involves the mass of the electron is really due to the existence of all the other particles in the world; whence he showed that it must have the form $\sqrt{N}/R$, where R is the radius of the Einstein world. Thus he obtained the equation

$$\frac{\sqrt{N}}{R}=\frac{mc^2}{e^2},$$

where m is the mass and e is the charge of an electron, and c is the velocity of light. Combining this with the equation

$$\frac{\kappa N m_p}{c^2}=\tfrac{1}{2}\pi R,$$

(where κ is the Newtonian constant of gravitation and m_p the mass of a proton), which is derived from the theory of the Einstein world, it is clear that N and R can both be determined. From R we can deduce by relativity-theory the theoretical expression for the limiting speed of recession of the spiral nebulae; Eddington's calculation gave 527·8 km. per sec. per megaparsec, which agreed fairly well with the observed recession, and thus provided a satisfactory check on the whole investigation.

The value of N can however be found in a totally different way. It is recognised in relativity-theory that a *measurement* involves four entities, namely two to furnish an observable relation, and two to furnish the comparison relation in terms of which the first relation is measured. Thus measured quantities are primitively associated with quadruple wave-functions. It is possible to calculate in spherical space the number of independent wave-functions of this form with the necessary relativistic property; and the total number of elementary particles corresponds to this number. The value so deduced is

$$N=\tfrac{3}{2}\,.\,136\,.\,2^{256},$$

which agrees closely with the result of the other calculations.

N is the "cosmical number" whose designation as "the number of particles in the universe" led to so much misunderstanding and scepticism. It is the number of particles in an Einstein world which is composed of hydrogen and which satisfies the requirements of quantum theory; these conditions define it completely, and different methods of evaluating it lead to concordant determinations. It enters into many physical formulae, *e.g.* it determines the ratio of the electrical to the gravitational force between a proton and an

* For simplicity we ignore complications due to degeneracy.

electron, the range and magnitude of the non-Coulombian forces between the heavy particles in atomic nuclei, etc. As an example of the way in which N relates astronomical to atomic constants, the following may be quoted. If k denotes the nuclear range constant (so the non-Coulombian mutual energy of two protons varies as e^{-r^2/k^2}), and s is the limiting speed of recession of the galaxies, then

$$ks = \frac{c}{\sqrt{3N}};$$

this striking prediction of Eddington's theory is verified with the best attainable accuracy by the observed values.

It may be proved in more than one way that the energy of the highest energy-level of the Einstein world is

$$m_0 = \frac{3}{4}\frac{\beta^{\frac{1}{6}} h}{cR}\sqrt{(\tfrac{4}{5}N)},$$

when $\beta = \frac{137}{136}$, $\hbar$ is Planck's constant divided by 2π, and c is the velocity of light. With this value of m_0, the masses of the electron and the proton may be shown to be the two roots of the quadratic

$$10m^2 - 136mm_0 + \beta^{\frac{5}{6}} m_0^2 = 0,$$

which gives

$$m_e = 9{\cdot}0924 \,.\, 10^{-28}\text{ gr.}, \qquad m_p = 1{\cdot}67277 \,.\, 10^{-24}\text{ gr.},$$

the observed values being

$$m_e = (9{\cdot}1066 \pm 0{\cdot}0032) \,.\, 10^{-28}\text{ gr.}, \qquad m_p = (1{\cdot}67248 \pm 0{\cdot}00031) \,.\, 10^{-24}\text{ gr.}$$

Just as Newton's discovery of universal gravitation was founded on recognising that the force causing an apple to fall from a tree was identical with the force restraining the moon in her orbit, and just as Einstein's discovery of general relativity was founded on recognising that the gravitational properties of mass are identical with its inertial properties, so Eddington's theory is founded on a belief that what are commonly regarded as distinct and mutually independent forces of Nature are merely different manifestations of the same force. This characteristic is well displayed in his treatment of gravitation. As we have seen, the proper-energy (which is the same as proper-mass) of an observed particle depends on the highest energy-level of the Einstein world. If, however, the system which is observed contains a large number of particles, the uppermost energy-levels would have become exhausted in supplying them, and it would be necessary to include some particles from deeper levels, which have a lower proper-energy. This fact, that the proper-energy of n particles is less than n times the proper-energy of one particle, is identified by Eddington with the phenomenon of gravitation, the difference in energy being what is ordinarily described as gravitational potential energy. *Gravitation is thus a consequence of the exclusion principle,* which forbids multiple occupation of energy-levels; but he goes beyond this, in linking gravitation and exclusion with two other physical principles of which we must now speak, namely *interchange* and *Coulomb energy.*

A distinction has long been recognised between two different kinds of chemical linkage; on the one hand, the "polar" or "hetero-polar" linkage (as between the sodium and chlorine atoms in NaCl), which is characteristic of salts, and on the other hand, the "homopolar" linkage (as between the two hydrogen atoms in the hydrogen molecule). The possibility of the latter kind of coupling can be traced back ultimately to the fact that electrons are indistinguishable from each other, so that the interchange of two of them does not affect the physical situation; *e.g.* the electrons belonging to the two atoms in

the hydrogen molecule can change places without causing any alteration in the molecule. Since the molecule thus has the same energy in two different "states", it comes under the dynamical category of "degenerate" systems; and we can show that, as a consequence of the interaction of the two atoms, there must be two distinct states of the coupled system, of which one will have greater energy than the two separated atoms and the other will have less energy. The former corresponds to a repulsion of the two atoms and the latter to a binding, which explains the existence of the molecule.

Eddington brought to the discussion of this class of problem an entirely new and original conception, namely the introduction of an angle θ such that the change from θ to $\theta + \pi$ corresponds to the interchange of two electrons. θ can then be treated as what is called in dynamics an "ignorable coordinate", and there will be a momentum conjugate to it which, as he showed, represents the dynamical consequences of interchange. This idea has proved of cardinal importance, for it has brought about the emancipation of physics from the bonds of space and time; the coordinate θ has no existence in space, and it leads naturally to Eddington's favourite notion of "extra-spatial flow of probability". By its means he succeeded in proving that Coulomb energy is essentially nothing but the energy associated with interchange, that interchange and exclusion are different aspects of the same ultimate principle, and that the reciprocal of the "fine-structure constant" has the exact value 137.

The design of reducing the number of independent physical principles is carried out very thoroughly, with the result that the qualitative theory assumed by Eddington as the basis of his researches differs considerably from that most generally accepted at the present time; for instance, he refused to believe that the meson observed in cosmic rays is identical with the theoretical meson introduced by Yukawa to explain the forces between the heavy particles in the nucleus; indeed, he rejected the Yukawa particle altogether, reserving the term "meson" for the cosmic-ray particle, which also is explained without introducing fresh conceptions. His theory of the nucleus, again, was distinctive; from general principles he derived two different types of binding between a proton and an electron, one the ordinary binding by a Coulomb force, and the other a "co-spin" binding, which can unite the proton and electron into a neutron; the nucleus he conceived not as an aggregate of protons and neutrons, but as an aggregate of protons and electrons, the latter being united by co-spin binding, not to the individual protons, but to the whole aggregate of them.

The theory is closely associated with, and to a great extent dependent on, a mathematical symbolism which Eddington created and to which he gave the name of the *wave-tensor calculus*. The fundamental ideas which led him to its construction may be traced back to three main sources which must now be referred to.

First, there was the calculus of quaternions, which had been discovered by Hamilton in 1843. The essential point of quaternions is that it enables geometrical properties in ordinary space to be stated and proved by means of a non-commutative algebra with four principal units; if these units are denoted by 1, i, j, k, then the fact that a coordinate x is measured parallel to the x-axis is represented by associating x with the symbol i, so the three units i, j, k correspond to the distinct *natures* of the three spatial directions, and a vector whose projections on the coordinate axes are x, y, z, is represented by

$$ix + jy + kz;$$

in order to discuss the properties of vectors it is necessary to introduce *quaternions* which are of the form

$$w + ix + jy + kz$$

where w, x, y, z are ordinary numbers, and to suppose that the multiplication of the symbols is conducted according to the rules

$$ij=k, \qquad jk=i, \qquad ki=j, \qquad ijk=-1.$$

With these dispositions, we find that geometrical relations can be very simply described; *e.g.* the vector which is obtained by rotating a vector ρ (conically) round a unit-vector ϵ through an angle ω is

$$q\rho q^{-1}$$

where q is the quaternion $\cos \tfrac{1}{2}\omega + \epsilon \sin \tfrac{1}{2}\omega$.

Secondly, there was the geometrisation of physics which had been achieved in 1915 by Einstein's theory of General Relativity. All observations are essentially observations of coincidences of two or more points in space-time and thus are *topographical* relations; our knowledge of the external world consists simply of the rules governing these topographical relations, and thus physics may be regarded as absorbed into a generalised type of geometry. So the idea presented itself, to discover a type of analysis which would bear to this geometricised physics the same relation as quaternions bears to the ordinary geometry of three dimensions; in short, to reduce physics to non-commutative algebra. This became Eddington's purpose, and in realising it he was helped by the third source of his ideas, namely the quantum-mechanical principle that *every physical quantity can be represented by an operator*: the most familiar example of this is the use of the operator $\dfrac{h}{i}\dfrac{d}{dx}$ in wave-mechanics to represent the momentum conjugate to a coordinate x.

What is sought for, then, is a non-commutative algebra with units E_1, E_2, E_3, . . . , analogous to the quaternion units i, j, k, and such that our knowledge regarding, say, an electron, can be represented by expressions of the form

$$\sum_{\mu} E_{\mu} p_{\mu}$$

where the quantities p_{μ} are ordinary numbers. The E_{μ}'s represent the *nature* of the physical quantities, and we may expect that four of them will be associated respectively with the directions of the axes of space and time (or what is the same thing, with the directions of the four-vector of linear momentum and energy), that six of them may be associated with the components of the six-vector (in space-time) of angular momentum, that four may be associated with the four-vector of magnetic moment and energy, and that two more may be required to represent electric charge and magnetic pole-strength; altogether 16. This is, so far, somewhat conjectural; but it so happens that the non-commutative algebra which is perhaps the next in order of simplicity to quaternions has precisely 16 units. This, which is called *sedenions*, may be described as the square of quaternion algebra; to obtain it, we take the units of ordinary quaternions

$$1, i, j, k,$$

satisfying the usual relations $ij=k$, etc., and take also another set of units of ordinary quaternions,

$$1, i', j', k',$$

where $i'j'=k'$, etc., making the rule that the second set commute with the first set, so that $ii'=i'i$, $ij'=j'i$, etc.; then an expression involving all these quantities will be a function of the 16 linearly-independent units

$$1, i, j, k, i', j', k', ii', ij', ik', ji', jj', jk', ki', kj', kk'.$$

Eddington takes these to be his E_{μ}, and asserts that *the sedenion algebra obtained is the desired representation of the structure of the external world.*

The most convenient notation is to take not unity but $\pm\sqrt{-1}$ as the first E_μ, which Eddington denotes by E_{16}, and then to represent each of the fifteen others by a double-suffix notation $E_{\mu\nu}(\mu, \nu = 0, 1, 2, 3, 4, 5)$, with the convention $E_{\nu\mu} = -E_{\mu\nu}$; their properties may then be represented by the multiplication-table

$$E_{\mu\nu}E_{\mu\nu} = -1$$
$$E_{\mu\nu}E_{\nu\sigma} = -E_{\nu\sigma}E_{\mu\sigma} = E_{\mu\nu}$$
$$E_{\mu\nu}E_{\sigma\tau} = E_{\sigma\tau}E_{\mu\nu} = E_{16}E_{\lambda\rho}$$

where $\mu, \nu, \sigma, \tau, \lambda, \rho$ is an even permutation of 0, 1, 2, 3, 4, 5. This multiplication-table defines a *group-structure*, the $E_{\mu\nu}$ being spoken of collectively as a *frame*. The E-frame is brought into physics by identifying the four-space of the symbols $(E_{15}, E_{25}, E_{35}, E_{45})$ with space-time, so that if (x_1, x_2, x_3) are the ordinary spatial coordinates of a particle, and t the time, the position-vector is

$$E_{15}ix_1 + E_{25}ix_2 + E_{35}ix_3 + E_{45}t,$$

(i being $\sqrt{-1}$, and arising as in relativity-theory from the distinction between space and time). Eddington shows, however, that the complete position-vector must be taken to be

$$E_{15}ix_1 + E_{25}ix_2 + E_{35}ix_3 + E_{45}t + E_{05}ix_0,$$

where x_0 is a real space-like coordinate which cannot be transformed into any of the space coordinates (x_1, x_2, x_3) by a real relativity-transformation, so that x_0 is not in ordinary space; actually, x_0 plays in this theory a part resembling that played in relativity-theory by the "radius of curvature of space"; this will seem not unnatural if we remember that when space-time is regarded as a hypersphere immersed in a five-dimensional Euclidean space, the radius of curvature is pictured as a space-like coordinate in the fifth dimension normal to the hypersphere. It is a remarkable instance of Eddington's success in relating cosmical theory to atomic theory that this coordinate x_0, connected originally with the curvature of the universe, plays a leading part in calculating the metastable energy-levels of the hydrogen atom.

The symbolic E-frame is thus "anchored" in ordinary physical space, but transcends it; ordinary space and time are merely four of the sixteen dimensions of physical reality. Eddington has indeed gone further than any other investigator in getting rid of the conception of Newtonian space and time as the stage on which the drama of physics is played by Newtonian particles; thus in his work on the hydrogen atom, instead of picturing it as an electron circulating round a proton like a planet round the sun, he has devised a totally different representation, analysing the atom into an "extracule" and "intracule" which may be compared roughly to the symmetric and skew parts of the wave function. *Where* and *when* have been reduced to something like the lesser status which they occupied in the Aristotelian-scholastic cosmology.

In any type of natural philosophy there is an obligation (not always recognised) to give some account of the distinctness and *individuality* of the elementary constituents of which the world is composed. Why does electric charge exist only in definite amounts—the charge of an electron—and why is it always associated with definite masses, such as the masses of the electron and the proton? Eddington's theory provides, for the first time in history, an answer to these questions. Since the whole physical description of a particle is carried by the E-number which represents it, we may expect that there will be some mathematical characteristic associated with the E-number of a truly *elementary* particle such as an electron. This characteristic proves to be one of amazing simplicity; it is, that the E-number—call it S—satisfies the equation

$$S^2 = S$$

so that every power of S is equal to S itself; it is *idempotent. Individuality is the same thing as idempotency*; such is the principle by which Eddington has resolved the universe into its ultimate constituents.

In formulating the quantitative results of the theory, it is to be remembered that experimental measures are expressed in the three traditional units gram-centimetre-second, which have no relation to any theory, and it was therefore necessary for Eddington to select three measured quantities to be used as conversion constants; he chose the velocity of light, the Rydberg constant for hydrogen, and the Faraday constant for hydrogen. Combining these with his theoretical determinations of the purely numerical constants of Nature, he obtained values for the charge of the electron, the Planck constant, the constant of gravitation, the speed of recession of the galaxies, the forces between protons in the nucleus, the masses of the electron, the proton, the neutron, and the cosmic-ray meson, the mass-defects of deuterium and helium, the separation-constant of isobaric doublets, the life-time of the cosmic-ray meson, and the magnetic moments of the hydrogen atom and the neutron. In practically all cases the values so found agree with the observed values within the margin of observational error. It was not without justification that in his latest writings, in which the work is presented as a unified whole, he adopted for it the name *Fundamental Theory*. It cannot be derived by logical deduction from existing physical theory; nor did Eddington ever claim that it could be. We must take it as he offered it, as a doctrine original in its deepest foundations, a new isomorphism between Thought and Nature.

Lastly we may refer to the series of publications in which the theory was developed. In the first paper, which appeared in December 1928, he asserted that the fine-structure constant e^2/hc must be the reciprocal of a whole number, which was determined in a second paper (Feb. 1930) to be 137. At the same time he began the development of the wave-tensor calculus. The cosmical number N, the ratio of the masses of the proton and electron, and the ratio of the gravitational to the electrostatic force between a proton and an electron, were found not long afterwards. In 1936 these results were presented systematically in his book *Relativity Theory of Protons and Electrons*, which was based almost entirely on what he called the "spin extension" of relativity-theory. After this he approached the same problems by a different method, which he called the "statistical extension" of relativity-theory; an account of this was given in his Dublin lectures of 1942, *The Combination of Relativity Theory and Quantum Theory*, published in 1943, in which he showed that the new principle permitted the calculation of all the fundamental physical constants except the cosmical number. In the last months of his life he planned, and almost completed, a book in which both methods were combined, and which was intended to replace all his previous writings on the theory; this is now being printed by the Cambridge University Press.

E. T. W.

GLEANINGS FAR AND NEAR.

1462. "Max loves to be tested. If your sister thinks she can defeat him, she is making a mistake. The cleverest mathematicians in Germany have all tried—and failed." . . . Arline was now thoroughly alarmed at the prospect of posing as an authority on mathematics, and Kirkwall racked his brains to set something which looked like the genuine work of a mathematician. Finally he prepared two tests, complicated multiplication sums with several square and cube roots.—Goodchild and Roberts, *The Prisoner's Friend*, p. 222. [Per Prof. E. H. Neville.]

TECHNICAL MATHEMATICS.*

Mr. H. V. Lowry (Woolwich Polytechnic) :

Mathematics in Technical Colleges.

Dr. McLachlan and I felt that it would be best to commence the discussion by dealing with the work done in technical colleges and thus lead up from the younger to the older. Mathematics classes in technical colleges can be said to fall into three groups : first, for those in which there are junior and technical schools there are mathematics classes for lads aged from 13 to 16 ; secondly, there are classes for those over 16 which we may group under the heading "Degree Courses ", that is to say, Matriculation, Intermediate and Final Degree and Special Honours classes for science or engineering; thirdly, there is what is probably the largest group, taking the colleges throughout the country, namely, the National Certificate classes leading up to the Higher National Certificate.

I have not had direct experience of teaching in a junior technical school except for about one-and-a-half years during the war while deputising for teachers who had been evacuated, but having discussed the matter with some of my colleagues it can, I think, be said that the Mathematics Syllabus in the junior technical schools is similar to the new Mathematics Syllabuses proposed for the School Certificate in the Report of the Joint Committee. That probably will have an important bearing on the development of the technical side of multilateral schools or of technical secondary schools after the war. I hope it will be possible, at any rate in the earlier years, to have a common course for the grammar and technical sides.

The mathematics done in the degree courses is similar to that in schools and universities because it is done for the same examinations, but it is taken a little quicker than in the schools : generally either full-time one year for each of Matriculation, and Intermediate, and two years, full-time, for Final. Or it may be done two years, part-time, for Matriculation (that is to say, one day or three or four evenings a week) ; also two years, part-time, for Intermediate, and three or four years, part-time, for Final. Even full-time, the speed is a little greater than at schools, but the part-time day or evening courses are much more intensive than anything ever done at school. Part-time students come one day a week or three or four evenings a week when they are doing Matriculation or Intermediate. You will therefore realise that the amount of time one gets for doing Intermediate Mathematics varies between six or eight hours per week, spread over thirty weeks. that is grouping two years together. This has to cover both pure and applied mathematics. Often the only full-time degree courses are run in a sandwich system ; that is to say, students attend the polytechnic, perhaps, for twenty-four weeks of the year for two years instead of taking a thirty-two week year with long holidays ; they are working the rest of the year. So, then, the degree courses often have to be done at a very much quicker rate than in the universities.

In the actual taking of the degree courses there is not much difference between what is done in a polytechnic and what is done in a school or university, except that the teachers have, by the very nature of things, a little closer contact with practical work, so that the examples they use are generally of a more practical nature.

I do not know the exact proportion of those taking degree courses to those taking national certificate courses for the whole country. In some colleges

*A discussion at the General Meeting of the Mathematical Association, April 5, 1945.

the courses are nearly all degree courses in the day-time; in others they are nearly all national certificate courses, but at Woolwich Polytechnic, which is, I think, fairly representative of the average, the proportion is about one taking Matriculation, Intermediate or Degree to three taking National Certificate courses, so that the national certificate mathematics looms larger than that in other classes. The Mathematics syllabuses for the courses in mechanical and electrical engineering, telephony, etc., are practically the same, except that one has a different type of examples in the different courses. We will, therefore, discuss them together.

The courses are all either one day or three evenings per week for about twenty-seven weeks, so that each year there is two to two-and-a-half hours mathematics for twenty-seven weeks. There are three years for the ordinary national certificate and two further years for the higher national certificate, mathematics usually being an optional subject in the last year. When the courses are voluntary, for instance in evening courses, the fall off in numbers is terrific—it is possible to start with, say, 200 or 300 students in the first-year class and only to have 60% in the second year, 40% in the third year and 10% to 15% in the fourth and fifth years. Probably a large part of the fall-off is due to students having difficulties with mathematics, because they cannot understand their other subjects without it. I personally do not believe that this is because mathematics is a harder subject than the others, but because it is taken too fast in the national certificate course, at any rate in London Polytechnics. If students do not get hold of the mathematics in the first year they are likely to drop off because they cannot understand the engineering subjects; so that much of the fall-off can be attributed to not getting hold of the elementary mathematics in the first year.

When attendance is compulsory, as it is in most part-time day classes for which firms let the lads attend for one day or sometimes one-and-a-half days a week, the fall-off is replaced by 50%, say, having to repeat the first-year course, the effect of which is not only bad for the students who have to repeat but very bad also for the newcomers. It has a cumulative ill effect.

These facts will have an important bearing on the new Education Act because, when attendance at school becomes compulsory up to sixteen, full time, and compulsory, part-time, from sixteen to eighteen years of age, we shall get larger numbers, probably, in the technical colleges from sixteen to eighteen and we are going to hold up the progress of the good students by keeping weak ones in the classes as we do now. The position in that regard is bad enough at present; it will tend to become worse unless we have the first and second-year national certificate mathematics taken at the technical secondary school, which would be the ideal way to connect up the national certificate classes with the new technical secondary school or the technical side of a multilateral school. We could then make sure that a student passed his first- and second-year mathematics, which in the second year is, I should say, about the same standard as the new school certificate. If the pupil did not pass then he would not be considered as suitable to go on to a national certificate course in a technical college; but if he passed he would have the great advantage that he would have his mathematics one or two years, at any rate, ahead of his engineering. Then we should not get that fall-off or that large repetition.

I will not now discuss in detail the branches of mathematics done in the first-, second- and third-year national certificate course except to say that, as far as I can judge, the immediate effect of Perry's crusade for the teaching of more practical mathematics was that empirical methods were used so much that there was a great waste of time for the student of average intelligence. To differentiate x^2 graphically is a waste of time for a student who can easily

follow the algebraic argument. Now that we have had time to try out the methods advocated by Professor Perry and others, I think the general opinion amongst mathematics teachers in the technical colleges is that the fundamental thing is to teach branches of mathematics that are going to be used during the next year or during the next six months in the engineering, *i.e.* to rearrange the order of the syllabus so that each part of the mathematics is used in some other subject within a year. This means, for instance, postponing simplification of fractions except those with single terms in the denominator from the first year of the national certificate till later, and at a later stage postponing partial fractions until wanted in calculus, and in calculus postponing the integration of all except the simplest functions till later. This rearrangement of the order requires most careful thought and I do not think we have reached anything like the ideal yet. Before a piece out of the usual syllabus is postponed we want to be quite sure that it is not wanted soon. To give an example, at one of the Board of Education summer schools at Oxford about six years ago a speaker on this subject said he thought that far too much time was spent on the solving of quadratics and the solution of triangles. The next lecture, a joint one between the mathematics and the engineering schools, was by Professor Cramp on the use of vector diagrams in electrical engineering, and in the first five minutes he used the formula for the solution of a quadratic equation and the cosine rule, on which we had just been told we spent far too much time! I am not saying that the empirical method should be rejected altogether but it should be held in reserve to use for a backward class, or student.

I have said hardly anything about advanced work, such as mathematics for special honours, and post-graduate courses for engineers, because the proportion of this work to the whole is very small and as yet, except in a few colleges taken throughout the country, comparatively unimportant. Dr. McLachlan is going to speak on the question of the post-graduate work for engineers and how we could develop it further, so I will not trespass on that ground, but I would like to mention one point in connection with this work. In the joint report of the Institute of Physics and the Mathematical Association the Committee advocate touching on a large number of topics rather than dealing with a few topics exhaustively, because it is felt that it enables the physicist to see the kind of mathematics that is going to be of use, and that as he has got to the graduate stage he will be able to follow it up himself. That has, I think, a bearing on the more elementary work. One of the great difficulties we have in doing intermediate work, part-time—I do not know quite to what extent this applies at schools—is that we have to do all the intermediate pure mathematics course in one year in two-and-a-half hours in the evening with perhaps a revision hour in the next year, and the physics department may be doing electricity in the first year at the same time that we are doing the pure mathematics, so that we cannot get in quickly enough the parts of mathematics that the physicists want; they are sure to want some calculus, the binomial theorem and so on in the first term. I am sure we need to think out some way—and this may possibly apply to schools as well—of dealing with the elementary work along the lines advocated for the advanced work in this report, *i.e.* to touch on a large number of topics which occur in intermediate mathematics in the first term and then go over them more thoroughly afterwards. I am not sure whether that would work well, but I think it something we have to try out.

This year, because of some difficulties of this kind, I tried to give our intermediate classes an extra hour in which I dealt with some of the things they were needing in physics in their first term, calculus, binomial and so on, which had not been reached in the ordinary pure mathematics course.

We have the same kind of difficulty in the national certificate course, for the engineers are always saying that they want logarithms, curves, elementary trigonometry, all in the first term of the first-year national certificate. It is absolutely impossible, of course, to get all the things they want in the first term. That may be righted when we have technical secondary schools because then there is the great hope of being able to get students up to the necessary mathematical standard before they come to the polytechnics to do engineering. Then they would be a year ahead at least in their mathematics and we should not have that difficulty.

As you probably know, just before the war a Technical Sub-Committee was set up to discuss the mathematics syllabuses in technical colleges. It was under the chairmanship of Professor Bickley and got as far as considering syllabuses of the junior course and first-year national certificate. It has not, unfortunately, met since about the first year of the war, and I fear that it is now going to have to meet without the help of Professor Bickley, who has had to resign from health reasons. Mr. Buxton, the secretary of the Committee, has asked me to ask you to thank Professor Bickley for all he has done on behalf of the Committee, especially in preparing some valuable notes on the Junior Courses and also in preparing a memorandum which was published in the *Mathematical Gazette* on the training of mathematics teachers for technical colleges.

I should like to conclude on that point. I do not believe that the average mathematical teacher in the technical colleges knows a great deal more about practical applications than the mathematics teacher in the secondary school. He goes to a technical college straight from the university ; he has to teach for twenty-two or twenty-four hours a week. He has to take, perhaps, every syllabus from a first-year national certificate to a degree ; perhaps he may have some classes in first-year national certificate, some second, some in Matriculation, some in Intermediate and some in Final. How he can be expected under those circumstances to read up practical applications, I do not know. To my mind the ideal way out of this would be—and I am sure by this means the mathematics teaching in technical colleges throughout the country would be improved enormously—if every mathematics teacher in the technical colleges was let off ten hours' teaching a week in the first two years and it was part of his duty to take the Final course in engineering in, say, two subjects, in those two years, doing practical work with the Final students. In a mathematics department with three or four lecturers in it, by sending them to different engineering subjects you would then, in the course of six or eight years, get one person in the department with authority on every branch of engineering taken in the engineering degree course. I am sure in that way you would enormously improve the teaching, because the practical applications would be given with knowledge and not, as now they have to be very often, by reading them up from books without first-hand knowledge.

The meeting then passed a cordial vote of thanks to Professor Bickley for his work on the Technical Sub-Committee.

Dr. N. W. McLachlan :

Post-Graduate Technical Mathematics.

1. *General Remarks.*

Originally this subject was called practical mathematics, being taught chiefly in Evening Schools. During the past fifty years, however, there has been steady progress, particularly since the end of the last war. This type of mathematics has no appeal to the mathematician, but is thrust upon him

because he has to earn his living. The standard in Britain falls well below that on the (pre-war) Continent and U.S.A.

There is still considerable divergence of opinion between engineers and mathematicians, as to the degree of rigour necessary, and the method of presenting the subject. This is mainly due to lack of sympathy of outlook, and calls for co-operation instead of isolation. It is also due to inherent disabilities, *e.g.* mathematicians seldom have a knowledge of engineering, while the average engineering lecturer knows little or no advanced mathematics.

It appears essential, therefore, that those who lecture on mathematics to engineers should be trained in both subjects and have some years of industrial experience. Also it is necessary that engineering faculties should have staff who know mathematics and are sympathetic with its application.

The mathematicians' viewpoint regarding the application of his subject to engineering is one which incites sympathy. Just as any true musician considers the orchestration of classical works in jazz or dance band form to be sacrilege, so any mathematician may be expected to feel strongly regarding the application of mathematics, devoid of rigour and style, to technical problems. The object of dance music, however, is to create an ephemeral diversion, whereas technical mathematics ultimately enables lasting benefits to be conferred upon the whole community. This is its justification, namely, its practical use for the amelioration of social services and conditions.

2. *Two Standards or Grades of Engineer.*

Mathematics is indispensable to the engineer, but many engineers have little or no aptitude for the subject. Moreover, we may divide engineers broadly into two categories: (*a*) those who have an elementary knowledge up to about inter-degree standard, (*b*) those who are able to assimilate and apply higher mathematics to practical problems. Much design and constructional work may be done by intelligent practical engineers in class (*a*). In class (*b*) there are post-graduate research workers at College and in industry. The standard of attainment of those in both classes would be improved appreciably by better methods of teaching and enhanced facilities, so that lecturers could know something about engineering.

It is realised, of course, that such proposals would involve considerable expense, but this would be justifiable in the interests of progress. The additional terms spent at College in attaining to a satisfactory standard of technical mathematics would ultimately be in the interest of national economy. Adequate time would be needed for the assimilation of mathematical knowledge to avoid its being merely superficial and, therefore, apt to be lost easily. At the moment the average college trained engineer loses a good deal of mathematical knowledge within twelve months of taking the degree unless he is in constant practice during employment, or has the good fortune to attend a refresher course.

It does not seem to be realised that after leaving College several years (thousands of hours) are needed for an engineer to become proficient at his job in industry. Nevertheless he is supposed to be proficient in mathematics after about 400 hours at College!

3. *Rigour.*

Solving intricate technical problems is not merely a question of getting what appears to be the correct result. The engineer should be able to demonstrate, within reasonable limits, that his findings are in accordance with his hypotheses; also that the latter are justifiable in practice. For this reason, in the earlier stages of his career the engineer should be encouraged to make checks and use numerical values, *e.g.* differentiating to reproduce an integrand, substituting a solution in a differential equation.

It is expedient to have an elementary knowledge of the theory of linear differential equations of the second order, and a working knowledge of convergence of infinite series and infinite integrals. The engineer should be aware of the consequences of uniform convergence, *e.g.* differentiation and integration term by term, representation of a continuous function. He should be acquainted with the conditions for differentiation and integration under the integral sign, inversion of the order of integration and use of asymptotic formulae. In other words more rigour is needed now than in the past, not for its own sake, but as a matter of practical necessity.

In this respect lectures based upon Hardy's *Pure Mathematics* and Bromwich's *Infinite Series* are necessary and would be beneficial.

In the past engineers have been lucky, for most of the infinite series and integrals encountered in applications were uniformly convergent. But as further progress is made in technical mathematics, the probability that this luck will hold becomes more slender.

4. *Syllabus.*

In compiling a syllabus of instruction, the various classes of engineer now existent must be borne in mind. As a short selection we have: aeronautical, acoustical, automobile, chemical, civil, communications (telegraphy and telephony), electrical (heavy), mechanical, radio, structural. There are certain topics which may be considered common to all branches of engineering. If the necessary alterations were made, the number of topics reduced, and practical applications included, a good syllabus could be compiled, from schedules B, C, of the report on "Teaching of Mathematics to Physicists", published in 1944 under the aegis of the Mathematical Association and the Institute of Physics.

In particular one may mention the following branches of mathematics: (*a*) Non-Linear differential equations—here it is essential that before any appreciable advance can be made, the pure mathematician must develop the technique which is now crude. (*b*) Contour integration leading to operational calculus. (*c*) The two and three dimensional wave equations: orthogonality: spherical harmonics: Bessel functions: Mathieu functions. (*d*) Vectors and Maxwell's equations . . . for radio engineers. (*e*) Matrices . . . for communications engineers. (*f*) Statistics . . . for engineers engaged in mass production of small parts, *e.g.* telephones. (*g*) Computational methods . . . for all post-graduate engineers.

5. *Applications.*

Problems involving various branches of Mathematics.

A brief list is given below:

(*a*) *Non-linear differential equations.* Sound waves of finite amplitude in loud-speaker horns: various mechanical problems where the restoring force in a vibrational system is non-linear: thermionic valve oscillators.

(*b*) *Contour integration and Operational Calculus.* Absorption and evaporation of moisture by porous materials, *e.g.* bricks, leather; aeroplane dynamics; annealing of metals; cable telegraphy and transmission lines; electrical circuits of various kinds; electrical wave filters; loudspeakers; microphones; radio and television receivers; refrigerators; torsional oscillations and various vibrational problems; impulses and transient phenomena in general.

(*c*) *Bessel functions of various kinds.* Vibration of circular membranes, microphones, loudspeakers and acoustics in general: electrical networks, transmission lines, radio receivers, skin effect in wires carrying radio frequency current, electromagnetic screening, eddy current heating, circular wave guides, electromagnetic resonators: vibration of bars of uniform and

variable cross-section, vibration of circular plates : viscous flow in circular pipes, motion of circular cylinder in fluid, vortex theory of screw propellers, interference in circular wind tunnel, ship waves ; heat transmission in circular cylinders.

(*d*) *Spherical harmonics.* Acoustical diffraction problems in connection with microphones, sound distribution at low frequencies from loudspeakers, electromagnetic resonators.

(*e*) *Mathieu Functions.* Vibration of elliptical membranes and elliptical plates, frequency modulation in radio telephony, loudspeakers, vibration of long column with periodic disturbance, sub-harmonics in vibrational systems, diffraction of sound and of electromagnetic waves, elliptical wave guides, eddy current loss in solenoid with metal core of elliptical section, vorticity due to fluid flowing past an elliptical cylinder, wave motion in lakes and canals.

6. *Proposal.*

To put the above views into concrete form, it is suggested that a report on " The Teaching of Mathematics to Engineers " be drawn up by representatives of the Mathematical Association and members of the leading Engineering Institutions, on the same lines as that already published, namely, " The Teaching of Mathematics to Physicists ".

Dr. H. A. Hayden (Battersea Polytechnic) supported the suggestion by Dr. McLachlan of the formation of a joint committee of the Institutions of Electrical and Mechanical Engineers and the Mathematical Association to draw up a mathematical syllabus applicable to the training of engineers and with a view to engineers and mathematicians working in closer conjunction. It was a healthy sign that the Institute of Physics had asked the Association to co-operate with them on the question of teaching mathematics to future physicists. Naturally, the Institutes knew what their students required and the Association should welcome any proposals on similar lines and be only too ready to try to meet the requirements.

As Dr. McLachlan had said earlier, all would deplore the fact that many students at technical institutes at present had to learn their mathematics in the evening after a hard day's work. It was desirable that vocational subjects should be taken at full day-time courses. Cultural subjects, such as were taken for their own enjoyment, might then be taken in the evenings, and it was hoped that engineers at the post-graduate stage would take mathematics for the enjoyment and satisfaction they got out of the subject ; it would be vocational in a sense, but not purely so. Dr. Hayden suggested that post-graduate work in the senior technical colleges might run on the lines of small groups of engineers who had interests in common. The engineer did not always know what he would be doing in six months' time ; he went from one problem to another. In view of the wide range which presented itself from radio to aeronautical engineering, it would not be possible to draw up a single syllabus to cover all the branches of the subject and get sufficient students from the area covered by any single technical college. Therefore he favoured small groups of students who would, if possible, come from the same works ; for instance, half-a-dozen from a radio works, who might all have the same type of problem to face, would bring them to the mathematicians at the technical college so that the problems could there be thrashed out ; the mathematicians would be able to advise the students as to what they should read and generally help them to cope with their problems. Dr. Hayden also had in mind short courses of lectures on advanced subjects lasting, possibly, six months, and he believed that by such means it would be possible to get into closer contact with those in factories and research institutions, and that

would go a long way towards the promotion of the teaching of technical mathematics.

Mr. W. F. Bushell (Birkenhead School) did not know whether the Mathematical Association preferred to speak of school mathematics or university mathematics, but his concern was school mathematics of the type to which Mr. Lowry had referred, and he had three points to make. Firstly, he was specially interested when he heard Mr. Lowry say that he made some attempt in his technical mathematics to make young students study problems connected with the particular job they would be taking up. He, however, thought that must be difficult, unless the technical college was exceptionally large, because it had to be remembered that there were in the colleges future technicians of various descriptions ; they all had to be catered for. Hence it would seem necessary to have some kind of general syllabus which gave the grammar of mathematics ; in elementary mathematics the grammar was necessarily extensive and some boys perhaps got little further than that. Nevertheless, it appeared from what Mr. Lowry had said that some real attempt was being made in the technical colleges to take quite young lads beyond the mere grammar stage and fit them for their future work whether telephonic or some other form of engineering. All this was good.

Secondly, Mr. Lowry had said that about 60% of the lads—and the speaker took it that attendance at classes was on a voluntary basis—failed to stay the two-years' course. That was of deep and grievous interest. The boys wanted mathematics because they were going to be technicians. Did they fail to stay the course because of sheer inability to learn the necessary mathematics or because the mathematics as taught them in a group was not suited to the particular work they were later undertaking? Possibly an affirmative answer would be right in both cases.

Thirdly, Mr. Bushell pointed out that the new Education Act had already come into being and he took it that the new technical schools would have to teach technical mathematics as part of the training given in the schools. But what, if anything, was going to happen to mathematics when it came to further education from sixteen to eighteen, or from fifteen to eighteen in the first place? Various suggestions had been made in regard to part-time education but nothing very authoritative had been said. He presumed that would be one of the tests of mathematics, because he did not suppose that it would be possible to interest youths between those ages, and give them what the law said they had to be given, unless those youths were offered something attractive to learn. It was all very well to say they had to attend schools, but it would not be easy to make them do so unless they came with good-will even though by law they were compelled to take part-time education. Hence the need for some attractive and useful syllabus with which to stimulate them.

Could Mr. Lowry say whether he was prepared to suggest anything for his technicians to cover the period between the years from fifteen to eighteen?

Finally, Mr. Bushell thanked Mr. Lowry for giving an idea of what was being done in the elementary technical colleges in regard to mathematics and hoped that Mr. Lowry would enlighten him on some of the points he had raised.

Dr. W. G. Bickley (Imperial College) felt all present were grateful to Mr. Lowry for pointing out, perhaps more by implication than explicitly, the difficulties under which many of the technical colleges, especially the junior technical schools, had to work. They had not much time, and in that time they had also to do much which ought to have been done earlier.

Dr. McLachlan had shown what an enormous sphere there was that could usefully be covered by those few among the engineers who were likely to be engaged in development and design. But there were at least two problems to be solved : mathematics for the million, as it were, and mathematics for

the few. At the outset it was necessary to make clear that those concerned were supposed to teach *mathematics.* Mathematics was an abstraction, but the effort to feed youngsters who were thinking of cars, aeroplanes and wireless sets on abstractions was obviously doomed to failure. All the same, these youngsters must be given, somehow or other, the opportunity of making their own abstractions. They had to learn the technique, but in the end it was necessary to aim higher : unless that was done it would not be possible to keep the youngsters interested. They must be enabled to like mathematics because it was good stuff as well as because they realised that it would help them to do what they wished to do.

In order successfully to teach mathematics to technical students a teacher must quite definitely have a fairly good technical background ; he must know at least enough to be able to use technical terms correctly and appositely, and to create at least the illusion that he knew quite a lot about engineering. His technical background should be such that by the time he had read a few articles and, if possible, attended a lecture or two by engineers on any topic, he would know enough to be able to cope with the inquisitiveness of the ordinary technical student.

When one considered carefully the list of applications given by Dr. McLachlan and sifted out of each the " pure "—and the speaker said he hated that adjective " pure " in this connection—the " pure " mathematics, the *abstract* mathematics of those applications, it would be seen in how many cases, for instance, differential equations of the second order came up. It was important that engineering students should realise that when they were learning how to solve such equations, then by merely changing the meaning of the symbols it would be possible to talk about all sorts of engineering and technical applications. The analogies which occurred all the way through would be most helpful to any teacher who liked to take advantage of them in creating and stimulating, not merely interest, but also understanding. It was possible better to understand the behaviour of certain electrical circuits by thinking also of a mechanical analogy.

Dr. Bickley added that one aspect of the matter on which he had touched rather delicately arose out of Dr. McLachlan's admirable suggestion that it would be good if the Engineering Institutions and the Mathematical Association could get together in the same way as the Institute of Physics and the Association had done, but it was important in that connection to bear in mind that the original invitation had come from the Institute of Physics. That body wished its students to do more mathematics, and had definite views as to the type of mathematics it wanted. He personally was not convinced that the Engineering Institutions had, speaking by and large, convinced themselves that they wanted much more mathematics or had clear ideas as to what sort of mathematics they wanted. He was himself firmly convinced that, as Dr. McLachlan implied, Great Britain was not, or had not been, getting the amount of technical mathematics it should have had, and which was necessary if the nation was to keep pace with modern developments. One had only to consider and compare the technical Press in Great Britain with that of Germany before the war, or France or the United States, to see that a really mathematical article had not had much chance in this country. This was, mainly, because engineers seem to wish to avoid, rather than to use, mathematics. The older basic branches of engineering grew up in the days when rule-of-thumb worked ; it was not now good enough. The younger industries had had the chance to develop themselves mathematically, but the habits of mind which had created the earlier state of affairs were still operative. At the same time, there was hope and things seemed to be on the move. It was possible to feel that the Engineering Institutions might, sooner or

later, wake up and ask for the assistance of the Mathematical Association. Personally, Dr. Bickley was a little doubtful whether, if help was offered by the Association before the Engineering Institutions felt they needed it, the offer would be accepted with any gratitude or whether anything would come of it. He hoped he was wrong and that the request for help would be made soon.

Mr. L. W. F. Elen (Coventry Technical College) thought the most striking fact about technical mathematical education was that there was so much diversity throughout the country. As an example, in the technical college in which he taught there was a separate mathematics department, but he doubted if this was general. Frequently mathematics teachers served under the engineering side and in some cases all the teaching in mathematics was undertaken by those who were basically engineers. That was a bad practice because engineers recruited from industry were not always the most suitable people to teach mathematics and often dealt with the subject more academically than was the practice in secondary schools. Although there was a tendency to overrate industrial experience, it would be generally agreed that technical mathematics could not be taught satisfactorily without some knowledge of industry. This could be achieved by training post-graduates for the work, just as teachers are now trained for work in secondary schools. Part of that training should be spent in industry.

A point which had come out clearly during the discussion, and one with which the speaker agreed, was that technical colleges tried to do far too much in too short a time. There was a continual endeavour to cram into the students a large amount of knowledge, much of which they were unable to assimilate. The solution in theory was easy, the amount of work could not be cut down so students must be allowed more time in which to do it. This would mean that the day off from industry now allowed to most part-time students would have to be increased to two days a week or even more. The best way to train engineers was not through the orthodox type of university course but by giving them, if necessary, a very lengthy course, during which they would spend part of their time in industry and part on more academic studies.

In adding a word on the National Certificate examination, which was the examination taken in most technical colleges, Mr. Elen reminded his audience that it was an internal examination. He thought that most teachers who suffered under School Certificate examinations would feel that internal examinations were all to the good, but this was very doubtful. When pressed for time there was a tendency to concentrate on the things one knew were going to be given in a paper and ignore those which were not coming; a tendency to stress, even unconsciously, certain parts of the syllabus, so that the students tried to forecast from the emphasis of the lecturer what was coming in the examination, sometimes with a great amount of success. Mr. Elen advocated that the National Certificate should be an external examination and that there should be a more or less uniform standard throughout the country. The present method of submitting the papers to an assessor did not bring about this result. The national certificate has purely local significance which may or may not be accepted by local employers. Mr. Elen felt that most of the objections to external examinations could be overcome if the syllabuses were drawn up to give a wide scope to the work of the teacher or lecturer.

As to the actual work taken in the syllabus, Mr. Elen thought a great deal more emphasis should be laid on finding approximate solutions. For example, engineers were likely to come across equations not always simple or quadratic, say, $2 \sin x = x$ and the approximate solution was all that was wanted. Inte-

grals which occurred could often not be expressed in finite terms and it was not possible to deal with all the different functions which might occur. Approximate methods of integration were however easy to teach and easy though sometimes tedious to apply.

In conclusion, Mr. Elen supported those who had urged that the committee set up by the Mathematical Association should get going again. There was obviously an enormous field to be explored. Technical education was largely a new subject and he was convinced that in a number of technical colleges a great deal of valuable work had been done which was unknown to the majority of teachers throughout the country. Any report issued by the Committee would not only be valuable to members of the Association but to all teachers of mathematics throughout the country.

Dr. A. D. Booth (University of Birmingham) felt that the suggestion that mathematics be taught at the same rate as engineering was not fundamentally right because a student should not be doing engineering of the type requiring, say, equations at the stage when he was learning elementary mathematics, presumably at fourteen years of age. The type of engineering young people came across at that age was marking up pieces of metal and so on; certainly nothing that required mathematical ingenuity. They should get to their Matric. or some equivalent standard before starting to use mathematics in engineering applications.

There was another somewhat interesting point as to whether mathematical lecturers ought to do a course of practical engineering. Ideally that might be all to the good; they might, however, spend a rather unpleasant time with the engineers. He had spent a short period in industry and had found that industrialists did not take kindly to mathematicians; they were put down in a corner and expected to stay there, not to come out and, as it were, annoy the industrialists.

As to Dr. McLachlan's point in regard to accuracy being important in industry, all would agree that it was most important to learning, but its importance to industry was not of such great magnitude because if industrialists would not support education they only had themselves to blame in the matter. While accuracy should be the aim, the speaker did not think a matter of £2,000 or £3,000 spent in vain should be allowed to cloud the issue. If several large firms went to the extent of losing such sums they would probably be more willing to allow their apprentices and junior technical staff to benefit from the advantages of further education.

In regard to mathematics in the United States and on the Continent the speaker thought the continental folk were good but most of the American graduates he had met seemed to be inferior mathematicians. In American technical journals one could read fine articles by specialist engineers but the rank-and-file American engineer, and certainly the rank-and-file American graduate, was not up to the standard of his equivalent in Great Britain. It would be admitted that electrical engineers nowadays got the limelight so far as mathematics was concerned in connection with radio and radio location, etc., but surely such things as stress analysis and aerodynamics had taken quite a large place in the modern engineering repertoire?

Did all trained *engineers* need to be really good mathematicians? Taking the statistical analysis personnel in a works, it would be found that the number of mathematicians was small; some had a smattering of knowledge of the subject, but those who had occasion to use real mathematics were evanescent. As a result of experience in an engineering works employing 15,000 he had found two individuals who knew any mathematics, and they had to solve, perhaps, half-a-dozen problems a year; the rest of the staff did not bother; they had no use for mathematics. If they wanted to design

anything they drew it to ten times the scale, measured up and reduced it, and they then got something they regarded as reasonable. That seemed to be one current method of designing: to do a large-scale drawing and hope for the best, allowing a safety factor of 500%! Therefore it seemed that a large number of the personnel in industry did not need great mathematical knowledge, but perhaps that did not apply to electrical engineers.

Contour integration was a most elegant method, but to introduce it properly would involve a lengthy course of instruction in complex variable theory. Dr. McLachlan had made with force the point that it was necessary to understand what one was doing so as to be able to check up on the results. Contour integration was one of the subjects in which, if one did not know precisely what one was doing, it was possible to make more mistakes than in any other branch of mathematics.

Mr. E. E. Ironmonger (Plaistow Secondary School) drew attention to the great value of interchange between secondary school teachers and lecturers in polytechnics. Secondary school teachers gained immeasurably from experience in teaching in a polytechnic. If actual teaching experience was not possible, knowledge of what was being done was desirable. There were many text books on National Certificate mathematics which it would repay secondary school teachers to study.

As a teacher in a secondary school the speaker urged the necessity of avoiding the mistake of thinking that because students were not themselves going to be post office engineers, for example, therefore it was not necessary to teach applications. Surely there was a general educational value if students realised that at least somebody used equations in the modern world. The future bishop could understand and appreciate quadratic equations as applied by the post office engineer.

Mr. R. A. Fairthorne (Farnborough) spoke as one of the minor consumers of those who came out of the technical colleges rather than as a teacher of mathematics. Taking it all round, he thought the best kind of mathematics to teach engineers was mathematics springing from something concrete. It was not possible to abstract something from nothing. Engineers should know enough mathematics to pick up or read up a particular mathematical technique whenever that was needed. Certainly that meant that they had to know rather more about certain branches, most of those enumerated by Dr. McLachlan, than of others. But unless one was using a particular technique every day one tended to forget it. Those called upon to do a job in a certain way should be capable of reading it up and tackling it reasonably well. That entailed good *basic* mathematics with rather enlarged experience or exercises on certain aspects such as differential equations in general.

The operational calculus held one or two mysteries for him, one being that no electrical engineer below a certain unspecified grade ever got the right answers when he used it; another being that a huge variety of not very closely related techniques were always lumped together as "Heaviside Operational Calculus". At present, operational calculus suffered from being fashionable.

Mr. Fairthorne agreed that the Engineering Institutions and mathematical teachers should get together and settle the best way of getting into the heads of engineers the mathematics that was necessary, but he pointed out that engineers who had been alive long enough to get to a fairly prominent position in the various institutions were three or four generations behind in mathematics.

He added that eight or nine years ago when he was solving vibration problems by operational calculus, he had always to spend a fortnight afterwards doing them again by the obsolete methods of Routh so that the working

might pass through the administrative levels and be accepted by those there. That probably would not be true these days, but it was true eight or nine years ago. One had to watch that those who called the tune did not also dictate the technique.

The only form of mathematics which everybody in an engineering works did use was arithmetic. Pretty well every human being had to add up correctly at some time or other—even shop assistants. It was deplorable when one considered the time and money wasted in a year owing to the incredibly bad methods so often used now. It was certainly disheartening to see an honours graduate in mathematics solemnly multiplying by 9 by turning the handle of a Brunsviga machine nine times. He had seen that done often. It would be much better to use a bead frame—an exceedingly efficient adding machine when properly used. In computing and in general arithmetic there could not be too much practice, whether students were training to be bishops or engineers.

Dr. McLachlan had read out a lengthy list of breeds of engineers differing mainly by nomenclature than by actual fact and mainly, of course, subdivisions of electrical engineering. Actually there were only three kinds of engineers : those who dealt with electricity ; those who dealt with materials that had little shear strength—fluids, gases, and the like ; and those who dealt with more or less rigid bodies. If one could get these three classes distinct from the point of view of teaching, if not from administration, it might be possible to achieve something. Production engineers might call themselves "Statistical engineers" tomorrow ; it was really only a matter of names.

Mr. K. J. Beaney (Battersea Grammar School) stated that he hoped that if a Joint Committee, as suggested, were formed, it would bear in mind the fact that there were rather limited needs for the great majority, but for the few the mathematical field was great and that the need for library reference books of suitable standard to satisfy the latter group was urgent.

Mr. J. F. Hudson (Aylesbury) supported the suggestion by Dr. McLachlan that steps be taken to get into touch with the leading Engineering Institutions, the Mechanical and Electrical, in order to frame a joint syllabus such as had been drawn up in co-operation with the Institute of Physics. It would not be advisable to wait until the Institutions themselves took the initiative. It was possible that the Association might wait a long time before being approached from outside. The Council of the Association should, therefore, give consideration to the taking of such steps as had been suggested by Dr. McLachlan. The initiative might profitably come from the Mathematical Association, and syllabus discussions would be likely to result in great benefit to mathematics and to mathematical teaching, as well as to the Institutions themselves. Mathematics was becoming more and more important in the different branches of engineering. Organised research in engineering had developed tremendously during the war. It was proceeding on a scale which would surprise most people, and many research engineers are using mathematics all the time. The more engineering research developed, the more important mathematics would become to the industry. Mathematics is a principal tool for engineers, especially for those engaged in research.

Mr. A. J. Hatley (Imperial College) underlined what Dr. Bickley had said, namely, that the physicists approached the Mathematical Association in the first place, and that the Association should not expect too much, in the initial stage, as a result of approaching the Institutions of Engineers. But the approach should be made, and to as wide an engineering fraternity as possible, not only to mechanical, electrical, civil, chemical and structural engineers, but to other engineering organisations.

As to Dr. McLachlan's remarks regarding the time devoted to mathematics in the graduate course, and the contention that the present 240 hours should become 600, the question was how to bring about that conversion, and that was a practical point which needed very carefully dealing with.

Dr. McLachlan had given a list of subjects, starting at non-linear differential equations and ending with statistics, and they all, of course, came into post-graduate work. He had, however, been told that there was hardly any place for elliptic functions in the galaxy. Could Dr. McLachlan say from his experience whether he could confirm that or whether there was a place for elliptic functions?

Mr. G. A. Garreau (Northampton Polytechnic, London) said that Jacobian elliptic functions were used in work on telephony.

Mr. Lowry, in replying, touched first on Dr. Bickley's point with regard to aiming high and Dr. Hayden's mention of students becoming interested in mathematics for the subject's own sake. As an example, Mr. Lowry said he had an evening revision class for final engineers and when he asked whether they would like to continue after the Easter recess they replied in the affirmative, with the proviso that he did what they wanted. That proved to be " any mathematics that has no application to engineering "! He thought the point was that through engineering mathematics those students had acquired an interest in mathematics for its own sake.

Mr. Bushell had spoken of the practical applications, and in this regard Mr. Lowry stated that he was in his opening remarks referring to those doing the National Certificate from the age of sixteen. He agreed that it was not possible to use engineering applications for young lads ; the only practical applications at the early stage were those such as finding the volume of a cylinder and so on, which could be understood without any knowledge of engineering. An application was not necessarily practical because it came from some branch of engineering which was about three years' ahead of what students were doing at the time ; they would not arrive at an understanding of engineering by that means.

As to the mathematics in the National Certificate examination being set internally, Mr. Lowry said that he and many others teaching mathematics in technical colleges felt it would be better if there was a national exam. He personally was against the Norwood Report so far as secondary schools were concerned, because he found the internal type of examination unsatisfactory. There was here a point as to which perhaps mathematicians also might have some say : so far as he knew, all assessors in mathematics in the National Certificate examination were engineers. A year or two ago he had had a question put as to calculating volume by integration and the engineer assessor wanted the students to explain how they got the integral, the adding up of the small elements and so on. The students had only been learning calculus for about five or six months. It had taken about two years for the calculus to sink into his own brain sufficiently to enable him to explain why he was doing this, that and the other. With a mathematician as an assessor that kind of thing would not arise.

Dr. Booth had said that mathematics would usually be ahead of the engineering. Of course it would for those taking the Matriculation and Intermediate courses, but in his opening remarks he was referring to the National Certificate course taken by boys at sixteen who knew practically no more than elementary arithmetic, although they might have been to an evening institute or central or junior technical school. It was found necessary in the case of those boys to do three or four weeks' revision of arithmetic, and then, almost immediately, they started wanting trigonometry in their engineering science and a knowledge of curves in engineering drawing, and they also

wanted logarithms ; so that with those students, at any rate, mathematics was not sufficiently advanced ; it needed to be one complete year ahead.

Mr. Lowry felt that engineers now had a far greater interest in mathematics than they used to have, and he believed that the institutions concerned would welcome an approach by the Mathematical Association. He did not agree that there would be resentment ; that kind of thing was dying out. One way might be for the committee already in being to ask each of the engineering institutions to nominate a representative to be co-opted on to the committee.

Dr. **McLachlan** agreed with Dr. Bickley that the approach to engineering institutions might not be received favourably, but the Association must not be discouraged. It was necessary to elevate technical mathematics in this country to as high a position in engineering education as it had on the Continent. If Great Britain was to keep pace with other countries, from the viewpoint of modern technical developments, technical mathematics must be well to the fore.

Mr. Lowry's suggestion that representatives of the various engineering institutions be co-opted on the existing committee was sound. It was necessary, however, that the institutions should nominate persons whose mathematical knowledge could be approved by the Mathematical Association. As Mr. Fairthorne had said, the leading lights of the institutions were probably about three generations behind the times in mathematics. Nevertheless, it must be realised that the representatives selected must know mathematics from the engineering viewpoint. Accordingly, the institutions should be approached, or engineers selected therefrom or from industry, who had an adequate mathematical knowledge.

Doubt had been expressed as to the standard of mathematics achieved by the Americans. One can only judge from output. The Americans had developed a large proportion of the modern mathematical and theoretical aspect of the subject of wave guides and had backed up their work with practical information confirmative of the theories. In a book he had reviewed for the *Mathematical Gazette* * there were about sixty references to various papers, of which twenty-nine were written by Americans and only three by authors in Great Britain. It was possible that the *average* American engineer did not know a great deal about mathematics, but Dr. McLachlan had no information in this respect.

As to mathematicians obtaining industrial experience, he had seen them in industry. At first conditions contrasted unfavourably with the academic atmosphere of a University and mental adjustment was difficult. After a time acclimatisation occurred and the mathematician realised that he was doing a good job of work when his knowledge was applied to the amelioration of social conditions, through engineering. Naturally, there were individuals who were unable to accommodate themselves to industrial conditions.

With regard to Dr. Booth's remarks on contour integration, it was the old story of trying to cover a course in 240 hours which ought to take 700 hours. Tuition in any branch of mathematics was ineffective unless it was done thoroughly. The subject matter had to be assimilated and many examples worked out before students fully appreciated the logic of the analysis. If the logic was faulty, mistakes occurred. As to turning the 240 hours into 700, lengthening the course would be necessary, but the difficulty was obvious. It might be helpful to get a committee together which would be strong enough to say precisely how the teaching should be done, and that at least 700 hours would be required in which to do it.

Apart from Mr. Garreau's remark that elliptic functions were used in

* Vol. xxvii, pp. 143–4 (July, 1943).

telephone work, Dr. McLachlan had no experience of them except in certain researches in non-linear equations. Unfortunately, those researches had been interrupted by the war.

Prof. E. H. Neville writes: Listening to this discussion, I was all the time expecting a reference to the obvious and fundamental distinction between what any individual needs to know and what he must be able to recognise as known. I use Bessel functions so seldom that I do not carry in my head the details even of the recurrences; I know a Bessel function or a Bessel equation when I see one, I can penetrate their more transparent disguises, and for any particular purpose I can find my way about the textbooks and treatises; in the last resort I can consult Dr. McLachlan or Dr. Bickley or Prof. Watson, not unintelligently. The technical student has heard from Dr. McLachlan a terrifying account of the mathematics he ought to know, and I want to reassure him. It is my knowledge of Bessel functions he needs, not Prof. Watson's, my knowledge of the theory of functions, not Prof. Littlewood's, my knowledge of Fourier series, not Prof. Hardy's, and it is far better for him to increase as much as he can the extent of mathematics that he knows in this sort of way than to pant hopelessly behind Hardy, Littlewood, and Watson in turn along their several roads. I am no advocate of superficiality; a man's knowledge of his own job can not be too thorough. But the engineer is training to be an expert in engineering. It may be that he will find himself one day engaged on a class of engineering problems for which it is worth his while to make himself expert in lattice theory or Diophantine approximations, but he can not anticipate this possibility by becoming an all-round expert in pure mathematics. Quick efficiency in first aid, clinical skill when a practical problem has been reduced to its mathematical essentials, should be the objectives of his training, for without these qualifications he can do nothing but follow a prescribed routine. The mathematical profession has its Harley Street consultant and its Brook Street clinic. The technologist should be encouraged to believe that the application of mathematics depends on the standard which he himself attains as general practitioner. Indeed, if he is described as a good St. John's man, is that a reputation to be shunned?

1463. Back to school! There is not much that I remember about my elementary school education anyway. What do I remember? Episodes like the Inspector calling in Standard II and asking what change you would have from £1 if you had spent so much and so much adding up to 14s. $6\frac{1}{2}$d.—or some awkward figure like that. Several got the amounts added up all right, and then failed at the last hurdle to do the subtraction right in their heads. Only one horrible little boy got the answer right—perhaps practice in the shop enabled me to do sums then which I am sure I couldn't do now.—A. L. Rowse, *A Cornish Childhood* (Cape, 1942), p. 111. [Per Mr. F. W. Kellaway.]

1464. I mentioned Mr. Maclaurin's uneasiness on account of a degree of ridicule carelessly thrown on his deceased father, in Goldsmith's "History of Animated Nature", in which that celebrated mathematician is represented as being subject to fits of yawning so violent as to render him incapable of proceeding in his lecture: a story altogether unfounded, but for the publication of which the law would give no reparation. Foot-note. Dr. Goldsmith was dead before Mr. Maclaurin discovered the ludicrous errour. But, Mr. Nourse, the bookseller, who was the proprietor of the work, upon being applied to by Sir John Pringle, agreed very handsomely to have the leaf in which it was contained, cancelled, and reprinted without it, at his own expence.—James Boswell, *Life of Johnson*, Everyman Edition, II, p. 13. [Per Prof. H. G. Forder.]

SYLLABUSES FOR EXAMINATIONS TAKEN BY SIXTH-FORM PUPILS.*

Mr. K. S. Snell (Harrow), in opening the discussion on the syllabuses suggested by the Cambridge Advisory Committee, said : Schoolmasters are apt to criticise heavily examinations and examination boards. We now have an opportunity of doing the reverse. A committee has been formed with the object of producing, and keeping up to date, an agreed syllabus for Sixth form work. This was brought about by the initiative of the Cambridge Syndicate, and in particular of Mr. Brereton, whom we are glad to welcome here to-day. I am sure I am expressing your opinion when I say that this Association welcomes the first report of this Committee, and is grateful for being offered permanent representation on the Committee. We must hope that other examining boards will consider the report with care, and will be prepared to come in line with it, for the advantage of any syllabus is so greatly enhanced if it can be one that is agreed by all boards.

There are some general considerations concerning this syllabus which I would like to stress. First, it is an examination syllabus and must not be taken to express any limit to what we want to teach. It thus suggests a necessary minimum which University teaching authorities can assume to have been covered. Many proofs are not required in this syllabus. But no sane teacher will on that account present his or her pupils with results without any attempted proof. Instead we are free to develop results in such order as we like, and, most important, we are free to make experiments, and to wrangle on the best methods. Thus an absence of proofs in a syllabus leaves more room for initiative among teachers.

Secondly, the syllabus only covers a two-year course. During the war our boys and girls have had to leave school at eighteen. This has limited the time available for specialisation and it is fair to say that most boys only have two clear years of preparation before taking a University Scholarship examination. Thus it is fairer that these examinations should be based on a limited syllabus so as to discourage the early passing of a School Certificate and premature specialisation. The pupil who has a third year may take a Scholarship examination in December, and he can well go ahead with further work beyond the syllabus, knowing that if he is taking the H.S.C. examination in July he will profit by knowing more than is demanded and that he will have made a start on University work.

Thirdly, the syllabus counters very strongly the tendency to over-specialisation in technical subjects. The mathematician does not develop the habit of general reading as part of his mathematical work. The report lays down that one-third of school time should be spent on subjects other than mathematics and physics. It is a debatable point whether the time thus available for English, History, Languages, etc. should be spent on work to be examined in H.S.C. or not. Scholarship examinations already have the machinery available in their general papers. Pupils need some incentive, either an enthusiastic teacher—or an examination. The former is clearly preferable, but the latter may prevent their attempting to escape these other subjects. I find that the boy himself often presents the chief obstacle to this general education, as he is apt to try to escape literary subjects on the plea that he needs more time for his mathematics.

There are four syllabuses suggested in this report. The Ordinary course, essentially including Pure and Applied Mathematics, provides a minimum for

*A discussion at the General Meeting of the Mathematical Association, April 6, 1945.

a scientist or for anyone taking mathematics as a subject for seven or eight periods a week. Further mathematics is really an extension of this either for the abler pupil to cover in the same time, or for the normal pupil in eleven or twelve periods. It should be covered by the good scientist or engineer. Higher mathematics is the course for the mathematical specialist, who is assumed to take one other main subject, Physics, or History, or Biology, etc. in the two-thirds of his time. The Subsidiary course was one of the most difficult to frame adequately as it can serve such different purposes. It may provide a course for the biologist, or the economist, or for the many who welcome the opportunity of investigating mathematics after School Certificate age, as subsidiary to their main work.

I will now consider a few details. The Ordinary syllabus is based mainly on Calculus and Mechanics, with the necessary Algebra and Trigonometry. The Calculus net is spread as widely as possible as regards ideas, with a minimum of manipulation. It stops short of methods of integration and that is why the differentiation of inverse trigonometrical functions, and the use of partial fractions is excluded. On the other hand exponential and logarithmic functions had to be included—no-one can be said to have done a course of mathematics without coming across e—and these functions need an introduction to series, and that is why the use of Maclaurin's series is introduced. In mechanics Simple Harmonic Motion is introduced to avoid too much stress on constant force. The ability to integrate very simple differential equations, by the separation of variables, will also help in this respect.

In Further mathematics the Calculus course is continued, for the benefit of the scientist, and can include some differential equations, 2nd order, and easy partial differentiation. As a new and essential idea complex numbers are introduced. If the syllabus seems long for the time allotted I can only plead that the committee envisaged easy and direct questions for examination purposes, and felt that it was vitally important to give breadth of idea rather than heavy manipulative ability.

This is a fitting moment to consider the treatment of geometry. I am the first to "shed a bitter tear" at so much omission. It was felt that the long analytical grind through each of the conics, including special properties of each, was a mistake. What is needed in the Ordinary course is the ability to use the analytical method referred to straight lines, angles, curves, tangents and normals, points of intersection and so on. In addition the use of parameters should be encouraged from the beginning. I would go as far as to say that at this stage points on curves should be given numerically or in terms of a parameter but never as (x_1, y_1).

For Further mathematics another problem arose. The course had to be suitable for technical students, but also for others. We are indebted to the ladies on the committee who raised the point that geometry is of special cultural value, and can produce great interest in a general mathematical course. Hence some pure geometry as well as further analytical work is included as an alternative to more advanced Calculus and Statics.

In Higher mathematics there is far less geometry than is sometimes included in a school course. In arranging a two-year course the axe had to be applied somewhere and projective geometry and more advanced analytical methods had little connection with other parts of the syllabus and so had to be sacrificed. Sufficient was kept to provoke interest in projective methods, and to allow teachers much freedom in the order of their analytical treatment. Further work in this subject can be undertaken at a University or by keen students in their last terms at school.

There are other omissions from the Higher course of work which the good mathematician can do at school. These include further applications of the

use of complex numbers to factorisation of $\cos n\theta$, x^n+1, etc., some Mechanics, such as general components of velocity and acceleration and their application to motion on a curve and orbital motion. But syllabuses do not deter the clever mathematician, rather they are a safeguard to prevent the more ordinary pupil being crammed, and being forced to over-specialise. It is our hope that these syllabuses have set a step in the right direction.

Dr. E. A. Maxwell (Queens' College, Cambridge) : I begin with a few general remarks before I proceed to the Syllabus itself. Perhaps I should first apologise for absence from the President's address yesterday, as I was engaged with examinations and could not get away. If I should happen to repeat any of his points, you will be glad of the corroboration ; if I should differ, you will be glad to have alternative points of view.

At the head of our discussion, I would stress the need for understanding and co-operation between teachers and examiners ; without that foundation, no syllabus can have any effect. I was very sorry to hear last year one or two comments which seemed to me to be both unhelpful and misinformed. The impetus for many of the present proposals has come from the Examining Bodies themselves, and it is unlikely that their examiners will not do all they can to carry those proposals out. Of course they will make mistakes ; who doesn't? But, believe me, the first person to find out that a question is bad is the examiner himself. I am now an examiner of many years' standing, and I say frankly that I am proud to have worked with such careful and conscientious colleagues. I know by experience that their aim is always to do full justice to all the candidates who pass through their hands, and that tricks and traps are seldom set deliberately.

Coming a little closer to my main topic, I should like to discuss a point arising out of our Cambridge Scholarship Examinations just concluded. I have had to interview a number of candidates in Mathematics, and I took the opportunity to ask them several questions, in particular how much time they had been giving to mathematics. The answers, were, on the whole, in very good agreement : about half-time up to the Higher Certificate stage, but almost full-time for Scholarship. Now these replies raise issues too deep for the time at my disposal, but I think I ought to make a few comments. You will see in the proposed Syllabus (p. 4) the opinion that " not more than two-thirds of the School teaching periods should be devoted to the main subjects of study " and, a little later, that " rather less than half of the total teaching periods should be given to mathematics, so as to leave time for another main subject (which might well be physics) within the fraction of two-thirds already mentioned." Is it too much to hope that all schools may decide to proceed by agreement to take this step?

We should notice here that a difference begins to emerge between the aims of a Higher Certificate Examination, which sets its seal on a standard of achievement, and a Scholarship Examination, which seeks its fulfilment in the future. Even though the same syllabus were used for both purposes, the type of question set might be very different. For example, I might set a scholarship question :

Find the ranges of real values of x for which the function $x^3-6x^2+11x-6$ is positive.

For a pass examination, on the other hand, it might appear instead in the form :

Sketch the curve $y=(x-1)(x-2)(x-3)$, and deduce the ranges of real values of x for which the function $x^3-6x^2+11x-6$ is positive.

In the first form, we leave the candidate to find out and initiate his own plan of attack. In the second, we test whether he can perform certain stated

operations. The same syllabus could cover both variants, but the art of the examiner (for examining, like mathematics itself, is both a science *and* an art) is to cast his questions in such a form that just those qualities which he wants to test in his candidates will tend to be revealed.

Let me, then, gather together what I have been saying, and reinforce my plea for co-operation. The greatest service that a school can do for a boy is to educate him soundly and send him for examination prepared *in the spirit which that examination envisages.* All-the-day-round mathematics then appears in its true light, as an attempt (which, indeed, may easily succeed) to convince the examiner that the candidate's work is unduly higher than it really is (I say " unduly " advisedly ; no-one objects to a little judicious boosting), and the tragedy is that the candidate's own best interests may be sacrificed in the process. I once had a pupil, a very successful scholar, who could make no headway ; I found that his whole time at school had been spent in working at scholarship papers and concentrating entirely upon the examination. Being an able boy, he was successful ; but he had no mathematical background, and was unable to tackle any sustained piece of reasoning when he came to the University. I quote him as an extreme example, but the moral is clear.

Now I know some of the difficulties which the schools are up against, and most of you know more ; and while a boy's whole career depends financially upon the results of an examination, the temptation to cram will be severe. The Government's enlarged State Scholarship Scheme should ease the pressure on the better pupils, but I fear that so long as awards are restricted to N pupils (as they always must be) then $N+k$ candidates will go all out to get them, where k is a pretty large positive integer.

This leads to my last point before I come to the proposed syllabus itself. I want to add a word or two to the paragraph " The Mathematical Specialist ", on p. 5. If the suggested organisation is adopted, one-third of the pupil's time will be devoted to subjects other than his main course. If all schools would agree to this, much good would follow ; for a teacher would not then feel that he had got to keep up the pressure for competition's sake. The question then remains whether this additional work should be examined. I suggest the answer : " For Higher Certificate, yes ; for Scholarships, no." For the aim of the Scholarship Examination is to pick out those candidates most likely to benefit by special study, whereas the Higher Certificate should be a fuller test of the candidate's general ability. It might be worth while considering whether both of these points could not be met by making the actual receipt of a Scholarship depend on the possession of a Higher Certificate in which the additional work had been tested.

Now for the Syllabus. I do not propose to go into much detail ; I have had my say on the Committee itself, and I would rather hear your comments. The work is designed to give as much freedom as possible to the schools, based upon a framework which can be administered with reasonable efficiency by the Examining Bodies. The points which I should specially like to know are (i) does the syllabus provide, roughly, the right amount of work for a two-year course? (ii) are the suggested alternatives, where they occur, roughly equivalent? If we can agree on these broad principles, the details of wording, or of exact selection of material, may safely be left to the bodies concerned. If, on the other hand, we feel that there is either too much or too little, then the sooner we say so, the better.

I should, however, like to say a little on the subject of Geometry, which seems to be having rather a thin time just now. In Ordinary mathematics (that is, Mathematics mainly for Scientists) much of the traditional work on Conics has disappeared, nor does it reappear very firmly in Further mathema-

tics. There seem to be two points of view about this. The Examining Body of which I have most experience (the Oxford and Cambridge Schools Examination Board) had no Conics in its Group IV (Scientists') Course a few years ago, but recently elementary Analytical Conics have been added. We sent a questionnaire to our schools last year asking certain general questions, and it appeared from the answers that several schools welcomed the addition, while others preferred a geometry syllabus of the type now proposed. What views has this meeting? Both attitudes can be justified, and the Examining Bodies may have to allow for each of them.

In framing the syllabus in geometry for higher mathematics, we found considerable difficulty in reaching a satisfactory form of wording. Our aim was to suggest an approach on modern lines, which can lead naturally to the study of projective geometry later on. But we felt that, for the two-year course which we were planning, most of the time had to be given up to metrical geometry, as we explained in the note on p. 10. My own hope is that it may be possible to go further than we dared to suggest in the report, and here, again, I shall be interested to hear your views. I think that pupils taking a three-year course should certainly be able to do much more.

In closing these remarks, which I have tried to keep to the broad general principles most suitable to a meeting of this kind, I would underline a point which I made earlier : a syllabus by itself has little meaning ; the important thing is the way in which it is interpreted by teachers and by examiners. I look on the proposed syllabus as a framework on which the Examining Bodies can build, without being tied to the particular points of detail. Indeed my own personal feeling, not necessarily shared by any of my colleagues, is that, as the work becomes more advanced, the syllabus should become more general and less detailed, and I like to feel that, in setting papers at Distinction or Scholarship level, I am not bound at every stage to hunt through a syllabus for authority. You will probably feel the same about teaching. The important thing is that, if this syllabus or any modification of it, commands general approval, then there will be a common background against which we can all work.

Finally (as those of you who have read Mr. Brereton's book would expect), the Committee is to remain in being ; and I am convinced that in this personal contact, and in the personal contacts made at meetings such as we are now attending, teachers and examiners can learn to work with a common purpose which a mere syllabus alone can never achieve.

Mr. J. L. Brereton : I feel honoured to be invited to take part in this discussion. We who are responsible for examinations are apt to feel that we must creep into the room on these occasions and hope nobody has noticed us. I have written a book suggesting that examinations are respectable, and I notice that one or two of the reviewers counter this idea very strongly, writing, for example, "Mr. Brereton moves in circles where the examiner is in the saddle." We do not want the examiner to be in the saddle ; we want teachers to be in the saddle in this matter of syllabus and the examiners to be their servants. Whatever good this Committee has done in drawing up these syllabuses (which, as has been stressed, will be continually modified), we really want to know now what the teachers think about the suggestions. I feel that this great Association has an important part to play in this respect. You as a body represent the teachers of mathematics in the country and we in this room and in the various branch meetings of the Association are in a position to plan our teaching as we want it. Many of us are doing our best to see that examinations shall not be a drag on the setting up of machinery which will enable the teachers to rule in their own house. In the past the examination system has made this very difficult. I am hoping that it will be possible to

achieve some state of affairs in which the teachers define the subject matter on which their pupils are to be tested.

I ought to say here that the person who has had most to do with the drafting of these syllabuses, and to whom I feel a great deal of credit should go, is Dr. Powell of Gonville and Caius College, Cambridge. He is with us to-day and I hope he will have something to say later on, because he knows the discussions which took place in committee over many points of detail.

Next I must stress that the Cambridge Advisory Committee is not representative of other examining boards. It is not claimed that this syllabus is any more than the view of the representatives of the Mathematical Association and other Teachers' Associations thrashing matters out with representatives of Cambridge University and of two Cambridge Examination Boards. I call your attention, however, to a paragraph on p. 1 of the Introduction in which we say that we have raised certain issues which we feel should concern other examining bodies as well; in particular, the question of the separation of pure from applied mathematics. There is a strong feeling in Cambridge that this separation would be a mistake, but I understand that there is, perhaps, as strong a feeling in the north of England, perhaps in industrial areas, that it is essential to keep these subjects separate. I hope that during the discussion attention will be given to that question. The Northern Universities and London, the two largest examining boards, hold that pure mathematics and applied mathematics should be separated in the sense that students may take one without the other. That is an important principle and this is an excellent body to express an opinion on it.

The idea underlying the title " Sixth Form Examinations in Mathematics " is that all examinations which are to be taken by Sixth Form pupils should be based on what is known to be taught in that form. It is a first attempt to define a reasonable syllabus on which to set any kind of examination. We have been in touch with the Admiralty, the Air Ministry and the Institute of Actuaries as bodies that are interested, and I hope that in the course of the next five years or so—I do not know how long one should say—we shall reach some sort of agreement that examiners are not free to set any questions they like in these examinations; that they are tied by what is taught in schools, so that the students can know that they have a fair chance in any examination without specially preparing for it. That at least is my view.

Now a few general remarks on the question of the mathematics specialist. It has often been said that the Cambridge Scholarship Examinations in particular are responsible for over-specialisation in science and mathematics. We on the Cambridge Locals Syndicate found that, if we attempted to reduce our syllabuses, the schools said: " It is no good your reducing the Higher Certificate syllabus because we shall have to teach our pupils more for the Scholarship Examinations."

I have had little to do with the drawing up of these syllabuses. I have been able to sit back and listen to what has been said and draw conclusions. The general conclusion I have come to on the very knotty question covered by the paragraph " The Mathematical Specialist " (p. 5) is that mathematics teaching, like all other teaching, is a matter of stages; that each pupil has to pass along a course from the simple to the more complicated; that an understanding of the complicated depends upon a sound grasp of the simple, and that you can define certain levels of knowledge. You could take as the first level, the level of the intelligence tests used for Special Place Examinations. That is, we will say, practically no formal knowledge of mathematics but quite a lot of understanding of some of the points dealt with in mathematics teaching. You could then take the School Certificate level. Next the Higher School Certificate level. I have found that teachers have no doubt as to what these

levels mean. A teacher will say of an examination question: "That's School Certificate level" or, "This is just not taught at the School Certificate level." There is a sort of atmosphere of knowledge which is accepted at those levels. Whether it is a right level or not, it is an arbitrary level. Some of us have been working hard to try to re-define the level to a slight extent at the School Certificate stage, to include some things and leave out others; to make a change. At any rate, these levels exist, and we have to accept, as part of the theory of education, that knowledge does go through a graded course and that levels tend to establish themselves. I put it to you that, for fair competition in any examination, it must be absolutely clear what the level of knowledge is; that the pupils, the teachers, and the examiners know at what level the competition is set. Otherwise unfairness arises.

The problem that faced the Committee was to reconcile the claims of the moderate boy well taught up to a high level of knowledge in higher mathematics and the bright boy whose knowledge is still at a lower level. What are the relative claims of these two to scholarships? The fashion to-day is to prefer the second. It may not be the practice of university examining boards, but the tendency in the country at the moment is to say that the bright boy who has not specialised is to be preferred. It is a difficult problem. Take, for example, an extreme case, so extreme as to be absurd. On the one hand, the boy with a good working knowledge of Higher School Certificate ordinary standard mathematics and on the other a boy of the same age who can get a very high score in an intelligence test but knows no formal mathematics, not even long multiplication. The second boy is badly equipped to be an engineer or a chartered accountant or anything else of the kind, although admittedly he is highly intelligent.

What we are looking for is not merely what the psychologists call intelligence, but something else—knowledge which has been put together with great pains and handed down to us by our ancestors. It provides us with patterns of thought which we acquire with much difficulty. The teaching profession exists to select and teach what is valuable and to discard what can safely be omitted. The knowledge of mathematics gained by the boy who has reached ordinary standard in these syllabuses is of use to him, not only at the beginning of his engineering career, but as giving him ways of thought, and ways of dealing with principles, which are absolutely essential in the modern world. I suggest we should get out of our minds the idea of intelligence or ability as a simple measure of a student. The psychologists have been dinning this idea of intelligence into us for twenty years and it is a terrible over-simplification. We have to recognise ability at a certain level of knowledge. We cannot compare intelligence and ability at a low level with intelligence and ability at a high level. Knowledge is something which has to be imparted to the child, it does not just arise out of him. I know it is not fashionable to talk like this. The common idea is that children are like little flowers which will grow perfectly of their own accord. This is just not true.

But this question is of more than theoretical interest. If in scholarship examinations we pitch our level too high, we play into the hands of the school which makes a business of employing teachers who do nothing but train a few boys in the higher mathematics. It is well known to most of you at least that there are many good schools with perhaps only one or two mathematics teachers who have to teach all through the school, and therefore have not the time to give to a few specialists. There are other schools in which there are teachers who do nothing, or practically nothing, but deal with specialists. Correct me if I am wrong. The problem of attaining fair competition between these two groups is probably one that we cannot completely solve. On the other hand, it seems to me that if we pitch the level too low, at the demand of

the school which is not well staffed, that cannot cope with the more advanced knowledge—then we are in danger of denying the whole purpose of higher education.

So I suggest that we have, somehow, to draw the line between two extremes; the awarding of scholarships or special places on a level of intelligence tests with no formal knowledge at all, and the use of very advanced syllabuses demanding much specialised knowledge. These syllabuses are an attempt to do something of that kind, and you will note that we have found it necessary to define some more levels. I believe that over the next few years we have to thrash out in practice which of the three levels mentioned in the syllabuses is the most important for sixth-form work in the secondary schools.

The **President** thanked the three speakers, especially Mr. Brereton, for a most valuable indication of what those concerned should devote their most eager and vigilant thoughts to. Before throwing the subject open to general discussion Mr. Tuckey felt he should call upon one person at least by name. As the demolition of the building should begin at the top it would be well, perhaps, to have the question of the higher mathematics syllabus demolished by Mr. Durell.

Mr. C. V. Durell (Winchester College) said that if his deepest feelings had been stirred—as perhaps they had—in regard to the syllabuses, they had been calmed by the exceedingly gentle way in which all who had so far spoken had asked the audience to say frankly what they thought. He was profoundly grateful for the opportunity to say something on the section dealing with higher mathematics.

But, firstly, arising out of Mr. Brereton's remarks the speaker doubted whether there were many—in fact, he wondered if there were any—schools in which teachers who dealt with scholarship work did not teach all through the school. Personally he had always had a certain amount to do with scholarship work and for the whole of his time he had taught in the junior part of the school. He believed that to be the only way of keeping touch with the mathematics of the school. Mr. Durell expressed profound disagreement with Mr. Brereton's theory of levels, and he hoped to make clear his reason for doing so in the course of his remarks on the report which was an illustration of the theory of levels.

With the general idea of the report itself, that general education was desirable, Mr. Durell strongly agreed, as he thought all present did. It stated that university teachers themselves believed that the pupils who came to them with a good general education, tended to do better than those who had spent all their time after School Certificate on mathematics. That was a result one would expect. Did Dr. Maxwell think that, at any rate up to the present, the university teachers had supported this belief by their actual practice? As an example, about which he had felt strongly for a long time, Mr. Durell cited the general paper in the scholarship examination. Very rightly, the candidate who failed to show a moderately reasonable knowledge of a second language (French, German, or whatever it might be) was put down a grade in his scholarship classification. He had repeatedly asked the Cambridge examination authorities to give weight to the general paper also in the matter of upgrading. He could not understand why that should not be done. If they believed in the merits and value of a wide general education, why were they not willing to say that a candidate who did distinguished work in the general paper deserved to be put up a grade? If there was a desire to encourage the schools to give a good general education, surely that was the way to do so.

He did not at all like the idea of saying it was necessary to distinguish between an examination syllabus and teaching syllabus. It was true that there

were a few schools in the country which could, and did, disregard the examination syllabuses, and they were very right to do so; but it was equally true, as was shown by discussions he had had with innumerable teachers concerned with sixth-form work, that at least in 90% of the schools of the country the teachers were dominated by the examination syllabuses. It was not true to say "We put this into the examination syllabus, but you are free to teach things outside it." Very few enjoy such freedom; the majority had to say "This comes into the examination syllabus; we must teach it; this does not come into the syllabus, so we cannot include it in our teaching syllabus though we would like to do so." If it were known that the university authorities were going to say that those who did distinguished work in the general paper would go up a grade on their mathematics that would exercise a tremendous influence on the teaching curriculum, and Mr. Durell contended—and he hoped what he was going to say would not appear offensive because it was not meant to be—that if it was the sincere desire of university teachers that scholars should have a good general education, then that was the way to secure it. That was his first main point.

Secondly, why was it that boys who had done very well in the scholarship examination sometimes came to an abrupt halt when they began their university careers? It could be attributed partly to general education, but that was not the whole answer. An important element in the answer was to be found in the kind of mathematics the boys had been doing in school. Had they been doing mathematics which concentrated on particular problems and blind alley work or had they been trained to think along general lines and appreciate the scope of general principles? Had they been dealing with ideas which foreshadowed university work? That would affect materially their progress when they left school and went to the university. The major problem to-day for those concerned with scholarship candidates was the bridging of the gap between school and university work. That gap would not be bridged by trying to teach topics which overlapped with university work. Teachers in the schools did not want to do that, and Mr. Durell felt sure university teachers did not wish them to do so. The gap would have to be bridged by bringing sixth-form teaching into line with the lines along which teaching was now running at the universities, because the way in which subjects were now being taught was different from what it was, say, twenty years ago. Algebra and geometry at the university were now quite different sorts of subjects in comparison with twenty years ago. It was up to schoolmasters to frame their courses in such a way as to bring them into harmony with the kind of work done in the universities. Mr. Durell thought the standard of sixth-form teaching at school was at present rising in a most remarkable way, judging by what he had seen and by what he had heard in discussions among teachers. The report, if accepted, was going to set the clock back twenty or thirty years. In fact, the report as far as higher mathematics was concerned was disastrous—and he was using the most moderate language when he said that.

He had specifically in mind the subjects of geometry and algebra because they were most affected by the new higher mathematics syllabus. There had been an expansion in these subjects, though perhaps it was more accurate to say they had changed in texture and this had made the syllabus too long—and it was not only examiners who said that but the teachers themselves—and a cut must be made. With that the speaker agreed. The ground covered in the syllabus did appear to be increasing all the time. But he disagreed with Mr. Brereton in the suggestion that the remedy was a level cut; that would be suicidal. He did not think there were levels of that kind. It seemed as if the Committee when approaching the question had decided that it was

desirable to make a 50% cut, and then discussed where they should draw the line, and drew it horizontally. It was as if they said: "Here is a complete tree; we have to prune it; how much have we to take off? Four-fifths. All right; we must just cut it down there." He agreed that it was necessary to make a cut but, if there had to be an analogy, let it be vertical, or much nearer vertical than horizontal.

The danger of a very restricted syllabus lay in the fact that many of the questions set in scholarship examinations would tend to be elaborations—futile, sterile, with no element of outlook value. Teachers would say: "A problem of this type will be set; the pupil must be trained to tackle it whatever its merits." Some might disregard it, but many would not do so simply because they felt they were not free to choose.

To take a specific example, problems of properties of the triangle, associated points of the triangle and circles were specially mentioned in the syllabus. Surely if there were such things as blind alley problems, they were problems of that character, and yet they occurred regularly each year in the higher certificate examination, and frequently in scholarship examinations. The character of the papers set for scholarship examinations at Cambridge were at an amazingly high level. Admittedly, examining was a highly-skilled job and those at present responsible for the Cambridge papers deserved the greatest admiration. They could improve them one grade more. After the examiner had set the paper he should look it through and ask himself whether this or that question was a blind alley question, and when he came to a question in regard to which the answer was in the affirmative he should cross it out. The papers would then be even better; but they were really so good now that it was not a matter of great importance. All would agree that when they were training a candidate for the scholarship examination—and they must, of course, put on a certain amount of final polish for such examinations—one of the most important things they had to do was to train the candidate to decide which questions to leave unanswered. The speaker contended that these were frequently the very questions the examiner should have omitted.

Turning to the syllabus in geometry, Mr. Durell expressed the opinion that from it had been omitted almost all that was most valuable and inspiring in geometry. Metrical geometry was, of course, necessary, but it was not the most important part. Duality was not mentioned and there was nothing about the use of line coordinates. There was nothing about homogeneous coordinates. By all means cut out all the homogeneous stuff about the length of axes and all the artificial metrical applications. Cut out also the elaborations on conics referred to principal axes which involve an extensive study of geometrical conics or special artifices. That was all dead wood. But why cut out the homogeneous coordinates? Even poor old Pascal had been cut out. It really was fantastic!

The **President** felt sure those present would like to hear the views on the Cambridge examinations of members associated with the Northern Universities Joint Board. Perhaps Miss Chamberlain could express those views?

Miss K. S. Chamberlain (Nottingham Co. S.C.) was not at the moment prepared to do so. She wished, however, to answer one of the questions put by Mr. Brereton. In the Northern Board higher school certificate it was possible to take pure mathematics as a main subject, or applied mathematics as a main subject, or pure and applied mathematics together as a main subject. Hardly any girls tried the pure and applied papers together. The subject was a little heavy. It was found that when girls went up to the university, they could not get Intermediate exemption on the mixed papers. It was for the universities to help teachers and to make it possible for the two subjects,

taken together, to be recognised. Most of those who took the Northern Board's papers were satisfied with them.

Miss E. Œ. Wolstenholme (Bridgwater) said her school was faced with the problem of recognition for Inter. and the London Examination Board had said they would accept the mixed subject pure and applied.

The speaker begged leave to descend from the high plane of Mr. Durell's geometrical contentions to the lower one of subsidiary mathematics. She felt strongly about the subsidiary syllabuses as they represent a standard capable of attainment by the arts specialist, and one which the latter should be encouraged to take for its cultural value. It was sad that mechanics should be alternative to a further pure mathematical subject, statistics. For arts students, particularly in the case of girls, the mechanical background was usually poor, unless taken in the mathematics course, and the speaker felt that to make a mathematics syllabus in which mechanics could continue to be neglected would be a tragedy; hence her plea that mechanics should be a compulsory part of the subsidiary syllabus.

Dr. I. W. Busbridge (St. Hugh's College, Oxford), speaking from the point of view of an examiner who had examined twice a year for nine years for the five women's colleges at Oxford, said that at first she had set questions which went far beyond the suggested syllabus. The result was that only one candidate in a year would answer these questions and would probably make a hash of them, so in despair she had cut out all such questions. She had found that candidates who did answer the more advanced questions always needed to be retaught the subject when they reached the university, because they had learned it in a hurried fashion. In the scholarship and entrance examination there were now three papers of a more elementary standard: the first on calculus and coordinate geometry; the second on applied mathematics; the third on algebra and pure geometry. Those were all taken by some scientists also, and for them the syllabus for "Further Mathematics" was approximately what she used. There was a fourth paper for mathematical candidates. That included approximately what was under "Higher Mathematics". These syllabuses, she considered, made a good working basis for examiners.

About the general paper, she said that credit was allowed for very good marks, but where a candidate got an alpha on a general paper and beta on mathematics, that candidate was never offered a mathematical scholarship because the candidate was probably reading the wrong subject. Such candidates, when they came up to the university, made only moderate mathematicians though they were admirable members of the colleges. Usually the speaker's mathematical scholarship students got beta or even gamma plus on their general papers. Where she found a good mathematician with delta, the candidate was usually turned down unless her interview caused a revision of opinion. Mathematicians generally did adequate general work; those who did good general work were usually inferior mathematicians.

The **President** was tempted to contribute a minor note as an appendix to what he had said in his presidential address, namely, that in order to understand what was *not* elementary mathematics it was necessary to study a syllabus for additional mathematics. He hoped it would not be taken that in order to ascertain what was *not* school certificate mathematics it was necessary to study the syllabus for higher certificate mathematics, because there appeared on the syllabus for Higher Certificate (ordinary) mathematics the items *linear equations* and *elementary properties of the quadratic equation*. Anyone who claimed that these were not elementary mathematics and thought that neither linear equations nor elementary properties of the quadratic could be set in school certificate would find himself in difficulties.

Mr. K. L. Wardle (Warwick School) said that the syllabuses were exactly what the scientists were looking for in conjunction with the mathematicians. He welcomed them. But he could think of few schools which could run simultaneous courses in Ordinary, Further and Higher Mathematics. Some of the difficulty would be overcome if the University Scholarship Examinations were transferred to the summer term of the third year; and as a suggested possibility Science Candidates might read (O) mathematics for the first two years and (F) mathematics for the third year. While Mathematical Physicists and Mathematicians could read (O) mathematics for the first year, (F) mathematics for the second year and (H) mathematics—perhaps only some for the Mathematical Physicists—for the third year. This would allow third year Scientists to read (F) mathematics along with the second year of the other courses, if they wished, and give the Mathematicians extra time to read outside the syllabus. As many boys do not read Additional Mathematics for the School Certificate, as some do not think of University careers till their second year, and as most become burdened with school responsibilities in the Sixth Form, the three years before Scholarship Examinations would prove an advantage both to the candidates and to school administration.

Mr. J. W. Ashley Smith (Henry Smith School, Hartlepool) feared he came from an area which had not yet appreciated the advantages of the Northern Universities Joint Board examination. His school took the Durham higher certificate in which it was possible to take pure and applied mathematics together, and the situation would be covered by the ordinary syllabus as set out in the report. It had been surprising to hear that there was supposed to be in the North of England a feeling against joint subjects. He had not lived long there but he had not come upon many teachers who strongly objected on that score. He had not much contact with girls' schools, although he did teach girls.

In regard to the higher mathematics syllabus, whilst more moderate in his language than Mr. Durell the speaker felt that the pruning might be carried unduly far. After all, the syllabus was intended for mathematics specialists. It was not desirable to set up a syllabus which could be taken by anyone who felt disposed to do so. The situation in many schools was that the only pupils taking the higher mathematics syllabus were those cut out for it. Therefore they could go a good deal further than scientists who took it, not because they were cut out for it. It was not reasonable to make a syllabus which was only twice as long as the science syllabus on the ground that it was going to occupy twice as much time.

It seemed a pity that the geometry had been cut down so much. The idea of transformations had been cut out, apart from inversion. The higher mathematics syllabus did not contain any hydrostatics. That was the one place where the syllabus diverged from the recommendations of the Institute of Physics report, in regard to what the minimum content of the higher certificate should be. It seemed unfortunate that higher mathematicians should not be allowed to tackle those amusing problems in hydrostatics.

Like many secondary school teachers, the speaker supposed he was really more concerned with the ordinary syllabus because most pupils would take it. He was not expressing general disapproval of the ordinary syllabus; on the whole, it was very good. Simpson's rule was in square brackets. Surely "Approximate integration by estimation of areas under graphs" should also be in square brackets? The speaker favoured cutting out all proofs for the theoretical part of the mechanics. It always seemed a waste of time to try to put in proofs which were relics of the day when there was an attempt to add the geometry. One was never sure where results were supposed to be experi-

mental and where not. The mechanics might be reduced a little by omission of certain theoretical proofs, but the speaker wished to see included something in the way of impact. If direct impact were put into the ordinary mathematics it might make the mechanics more complete.

Mr. K. J. Beaney (Battersea Grammar School) referred to the statement under the heading " Some Points of Principle " (p. 5) : " We are emphatically of the opinion that, when mathematics and its applications are studied, they should be regarded as a single subject, and that the insistence on the division into pure mathematics and applied mathematics is detrimental to good teaching." That seemed unwise. An applied mathematics syllabus could well be enlarged to include not only mechanics, but also subjects such as statistics and parts of physical chemistry where, as in mechanics, the content of subject matter is relatively small and the standard of mathematics relatively high. This tendency to expand would occur more freely with pure mathematics and applied mathematics divided as at present.

Mrs. E. M. Williams (Goldsmiths' College) spoke with regard to the particular problems of girl specialists in mathematics. In the past girls' schools had given rather more attention to non-specialist subjects than had boys' schools, largely because women felt very strongly the claims of life, apart from the abstractions of mathematics. The new principle that at least one-third of the time in school should be devoted to non-specialist subjects would not mean a very great change on the part of many girls' schools, but Mrs. Williams was sure that the great majority of her women colleagues would welcome it as a principle to be accepted by all schools, both boys' and girls'. But there had been another difference between boys and girls which had made it difficult for a girl going on to the university to keep up to the level of the man student in mathematics. Not only had the girl spent more time on other subjects and less on specialist subjects at school but she had usually had a much inferior training on the side of mechanics. When a girl went up it was again and again on the applied mathematics side that she had the greatest difficulty. When one talked it over with those in charge of the administration in girls' schools, it was at once realised that the pressure of the time-table made it difficult for the girls to spend sufficient time on mechanics to enable them to reach the same level as boys. It seemed that the change by which the pure and applied mathematics would be included under one subject was going to make it much easier for the girls to ensure that they did as much mechanics as the boys. There would not be the tendency to push the applied side out of the time-table and for the girl to find herself constantly at a disadvantage. From the point of view of those in girls' schools, Mrs. Williams felt that her colleagues would endorse the new principles as to the time to be given to non-specialist subjects and the putting of all branches of mathematics into one paper.

Mr. A. Barton (Blundells' School) supported Mr. Durell's plea in regard to the geometry syllabus for the higher mathematics course. The point as to boys who reached a certain stage in their mathematical learning was important, and there was another reason besides that given by Mr. Durell. The speaker could not give the explanation but it seemed an experimental fact that a boy could go so far and no further. It should therefore be the aim of the higher certificate examination to sort out the boys who would be able to go further. It was no use making it too easy ; the geometry syllabus, in particular, should include things easy to boys who would become mathematicians and hard to those who would not. Projective geometry was eminently a subject of that type. There had been at the end of the geometry distinction paper in the Oxford and Cambridge Higher Certificate and also towards the end, usually, of the geometry paper in the Cambridge scholarship, a question

on homogeneous coordinates, which had been extremely easy to boys who had a real mathematical understanding and which, presumably, must appear hard to others. It seemed that the geometry syllabus had cut out that type of question, or at any rate brought it to the minimum whereas it ought to be increased. The approach to projective geometry in the school was bound to be different from that at the university. A boy when doing it was not at the stage when he could regard things entirely from a logical point of view. He could not start by discarding all real ideas or take easily to the idea of the net of rationality, for example, and begin there ; and perhaps the reason why projective geometry was frowned on was that it often consisted in proving results by stunt methods such as projecting into a circle and then using what Mr. Durell had referred to as dead wood ; but there was plenty of scope for the opposite kind of treatment, that of generalising results by considering a projected figure of them, and that appealed to the boys. To draw a figure in which the line at infinity appeared as a perfectly respectable line on the paper, and put in the points I and J fired their imagination and provided material for examination questions.

De Moivre's theorem was already in the syllabus for further mathematics. There seemed no point in having it there and not using imaginaries in the higher mathematics syllabus so far as geometry went.

The trigonometry syllabus for ordinary mathematics spoke of the formulae for sin $(A \pm B)$, cos $(A \pm B)$ and tan $(A \pm B)$. There had been questions set in the higher certificate requiring proofs of these for angles of any magnitude. It seemed that questions of that type, which did occur from time to time, demanded a great deal of work with very little result. One could demand the same sort of proof in coordinate geometry. " Prove that the area of a triangle is the usual determinant when the points are in any position ", to be done probably by drawing rectangles. The speaker thought the question undesirable and that it should be relegated to the category of things that the boy should be shown at the discretion of the teacher and not asked to produce himself.

Dr. F. C. Powell (Gonville and Caius College, Cambridge) said, in defence of Cambridge, that Oxford was not alone in giving credit to high achievement in the general paper in the scholarship examination. In the scholarship group with which he was acquainted the performance in the general paper was taken into account in every case, and boys were promoted to a higher grade as often as they were depressed into a lower one. In fact, an order of merit was drawn up which took into account the performance in the examination as a whole, and when it was stated, as in the circular, that good work would be taken into account, that was true. Dr. Powell said he could not speak for more than one scholarship group; but there was no doubt that considerable weight was given to the general paper so far as that group was concerned.

As to the division into pure and applied mathematics, the difficulty, from the point of view of a university teacher, was that he sometimes had to deal with boys who had taken pure mathematics but no applied, or applied mathematics and no pure mathematics. When that happened one realised the grave defects of that particular way of dividing the subject. In either case the boys had missed out something which was essential to a proper understanding of the subject as a whole because there was no doubt that a study of applications of mathematics did assist the majority of boys in their appreciation of pure mathematics.

It seemed that the Committee must take note of the many remarks that had been made to the effect that the geometry syllabus in higher mathematics had been cut down too much. What the Committee had in mind was that although they recognised that homogeneous coordinates and tan-

gential coordinates could undoubtedly be taught at school, they were doubtful if they could be taught in the two years under the time-limits imposed upon teachers, particularly if, after the war, there were to be five terms only between school certificate and the higher school certificate, as suggested in the Norwood Report, and not more than perhaps fifteen periods a week on mathematics as advocated by the Committee. That accounted partly for the, perhaps, somewhat old-fashioned appearance of the geometry syllabus in higher mathematics. The same consideration of the time available had led to the making of statistics and mechanics alternative in the subsidiary syllabus. There did not seem time available to cover the whole thing and do the necessary pure mathematics that must go with those two subjects.

In omitting hydrostatics the Committee had followed the recommendation based on the 1937 investigation into the higher school certificate. Some on the Committee concerned in that investigation were present and could voice their views. It was recommended that hydrostatics should be omitted from the mathematics syllabus; that recommendation had been followed and it, apparently, clashed with the recommendation of another Committee. One could not have it both ways.

Mr. W. Hope-Jones (Eton College) drew attention to the fact that in the subsidiary mathematics syllabus the sine and cosine of $(A \pm B)$ appeared without the tangent, whereas on the opposite page (p. 7) the tangent was mentioned. That suggested that the first treatment of sine and cosine of $(A \pm B)$ should omit the tangent. That would be a mistake because it was the natural sequel to it, and when one came to set questions on it it was easier to find good questions on the tangent than on the sine and cosine. The step from sine and cosine to the tangent was an easy one.

Under ordinary mathematics (p. 7) the algebra began with progressions. Personally he preferred the Algebra book which had a chapter on series in general, a commonsense treatment of series which could not be reduced to only two kinds of series for which boys crammed a formula. It was all very well to say one did not teach a formula; boys learned it whether it was taught or not. To set commonsense questions on series for which there was no formula was of much greater value.

Why square brackets for "Centre of gravity" in the middle of page 8? That was somewhat surprising because it seemed so much easier and more elementary than some of the subjects above, which were not bracketed. Surely if there was danger of over-loading the minimum course, Centre of gravity could be put into it and one of the others be exiled?

While not wishing to enter upon a large discussion on matters of principle, the speaker much regretted Mr. Durell's anxiety to prune off the side lines, because so often they were where the real fun came in. Without the fun teachers were not going to make a really good job of mathematics for the pupils.

The **President** thought it impossible to stress too highly the difference between the two-year and the three-year period, to which several speakers had called attention. Some were accustomed to three years and were inclined to criticise because of that.

Mr. K. S. Snell, in replying, said that already Dr. Powell had answered many of the criticisms. In reply to Mr. Hope-Jones he thought "Centre of gravity" was put in square brackets because it did essentially involve the summation idea of integration which was not essential for the other parts of integration, and therefore in pruning it was felt that it might possibly be left till later. "Series" was something on which the Committee could not quite agree; it became "Progressions" as a last resort. The difficulty appeared to be that as soon as series were included examiners might set a series of complicated questions. He hoped it would be possible to take Mr. Hope-

Jones' advice and alter that. The subsidiary syllabus was a separate one, and that accounted for the difference in regard to the tangent.

In reply to Mr. Barton's question as to proof of the sine of $(A \pm B)$, in framing the syllabus the Committee had in mind that certainly no complicated proofs would be asked for. Although the omission of that proof was not directly stated, it was not imagined that a difficult case would be set.

Mr. Snell said he had been delighted to hear the outcry in regard to geometry because he personally wanted the geometry syllabus to be longer than it was. He remembered his disappointment when Chasles' theorem, for instance, disappeared off a statement he had put forward. As Dr. Powell had said, there would have to be reconsideration and, he personally hoped, a broadening of that syllabus. Unfortunately no one had suggested what should be pruned. That was something which would have to be gone into in more detail. As Mr. Hope-Jones had said, some boys found elementary properties interesting, and Mr. Barton had said they could be used in later work. That suggestion would be gone into seriously.

Dr. E. A. Maxwell had practically nothing he wished to add. He was pleased to see the reaction in regard to geometry which happened to be his particular pet subject. He had been disappointed when he saw it shrinking; nothing gave him greater pleasure than to feel it should come back again after all.

Mr. J. L. Brereton, in reference to Mr. Durell's difference with him on the question of levels, said he was a little out of his depth on the geometry syllabus. Some of the things had very curious properties when projected! Possibly Mr. Durell had overlooked the curious property of examination levels: that whatever kind of a section one made, it ended up by being what Mr. Durell called horizontal. That was not just a clever point. The Committee were sincere when they said they wanted to learn from the views put forward during the discussion as to where the syllabuses were wrong. They would like to limit the higher mathematics syllabus by the kind of section that was really approved by those who were teaching higher mathematics; but in a few years' time any new section would seem to be a horizontal section defined by text-book writers and others. The Committee were not desirous of making arbitrary divisions on present syllabuses; they wished at the same time to put in the things which were considered most suitable and to omit what was out of date and there leave it for the time being. They were grateful to those who made a contribution in regard to higher mathematics, and he hoped the Committee would consider whether the syllabuses could be altered.

The **President** said he had over-estimated the time that the openers of the discussion required for their replies, and he accordingly invited further " little digs " from those who wished to make them.

A Member asked if Mr. Durell could add a word or two on the elementary parts of the syllabuses.

Mr. C. V. Durell said that if he might " have another dig " it would not be at the elementary because there he had no criticism. He had, however, something to say about the algebra in the higher mathematics syllabus and here he again complained about the cut. Exclusion of recurring series seemed to him to indicate the wrong kind of attitude. Not that he valued greatly the importance of recurring series. What seemed to him important in the modern spirit of the teaching of algebra was the teaching of difference and differential equations side by side. One of the best examples of difference equations lay in recurring series and he would like to see recurring series in the syllabus for that reason. This was another illustration of the undesirability of a level cut.

Again, it seemed extraordinary to include in the syllabus the use of $f'(x)$

for discovering repeated roots and excluding Rolle's theorem with its application to the location of roots. The mean value theorem opens out a fruitful field of discussion in sixth-form work.

Again, what was meant by "simple convergent infinite series"? He did not want elaborate examples but he did want the possibility of the kind of question which the skilled examiner could set and which showed whether the pupil had understood the spirit of the idea; and it was the spirit of such ideas which seemed to him to be making sixth-form work so valuable nowadays.

Miss K. W. McIntosh (St. Paul's Girls' School) said no one had lamented the absence of pure geometry from the subsidiary syllabus. For the benefit of those who took the subject for its cultural value she wished to see some pure geometry as an optional subject.

Mr. R. Sibson (City Boys' School, Leicester) said a few years ago he had had an opportunity of starting a sixth-form statistics course with an entirely non-mathematical class, consisting of boys who were not doing mathematics other than the statistics concerned. In the course of two terms he had succeeded in covering the ground indicated by the first three lines of the syllabus in subsidiary mathematics on p. 7, in two periods of 45 minutes each a week. That would be, if anything, rather more than adequate to cover the total syllabus quoted for the statistics, and it seemed that it would provide a reasonable alternative to the mechanics.

Miss P. M. Pickford (St. Swithun's School, Winchester) suggested that the subsidiary and ordinary syllabus should include some solid geometry (specially useful for those taking architecture) and that astronomy should be an alternative to statistics for the benefit of those taking geography or arts subjects.

Professor P. J. Daniell (University of Sheffield) said that he wished that our teaching of elementary geometry could follow Continental methods, where the geometry was woven in a pattern which uses the elementary notions of transformations of the various types which form groups. The abstract theory of groups was, of course, not possible at that stage.

Mr. T. D. Morris (Charterhouse), referring to the point raised by Mr. Sibson, felt that the subsidiary syllabus in statistics was quite a small one and could be got through easily. He had been doing similar work with a small group who had the school certificate but were not good enough to specialise. If homogeneous coordinates went back into the syllabus he hoped it would be at the level of the Cambridge and not the Oxford papers in which he felt there was a labouring of the methods in a way that should never be attempted in schools. He supported the inclusion of projective methods, but again at an elementary level.

1465. At that time the leading person about the Lakes, as regarded rank and station, amongst those who had any connection with literature, was Dr. Watson, the well-known Bishop of Llandaff. This dignitary I knew myself as much as I wished to know him; he *was* interesting; yet also *not* interesting; and I will speak of him circumstantially. Those who have read his Autobiography, or are otherwise acquainted with the outline of his career, will be aware that he was the son of a Westmoreland schoolmaster. Going to Cambridge, with no great store of classical knowledge, but with the more common accomplishment of Westmoreland men, and one better suited to Cambridge, viz. a sufficient basis of mathematics and a robust though commonplace intellect for improving his knowledge according to any direction which accident should prescribe—he obtained the Professorship of Chemistry without one iota of chemical knowledge up to the hour when he gained it.—Thomas De Quincey, *Reminiscences of the English Lake Poets* (Everyman's Library Edition, p. 54). [Per Dr. H. Lowery.]

MATHEMATICAL NOTE

1842. *On Note* 1727 : *Extension of Simson's line* (*Gazette*, **XXVIII**, p. 107).

Mr. Gibbins might be interested in G. A. Srinivasan's Note 605 (*Gazette*, XI, p. 20, January, 1922). My pendant to this note on p. 87 of Vol. XI was unfortunately wrecked by a misprint ; I give its substance in 1 below. For a figure, see Note 1727 (July, 1944).

In 2 and 3 I give a geometrical proof of the theorem about the join of the orthocentres.

1. Draw the chord PS of the circle ABC perpendicular to BC ; then AS is parallel to the Simson line of P for the triangle ABC. But

$$\angle PSA = \angle PCA = 180° - \angle C'CY = 180° - \angle C'XY = \angle B'XY,$$

and since $B'X$ is parallel to PS, AS is parallel to XYZ, that is, the Simson line of P is parallel to XYZ.

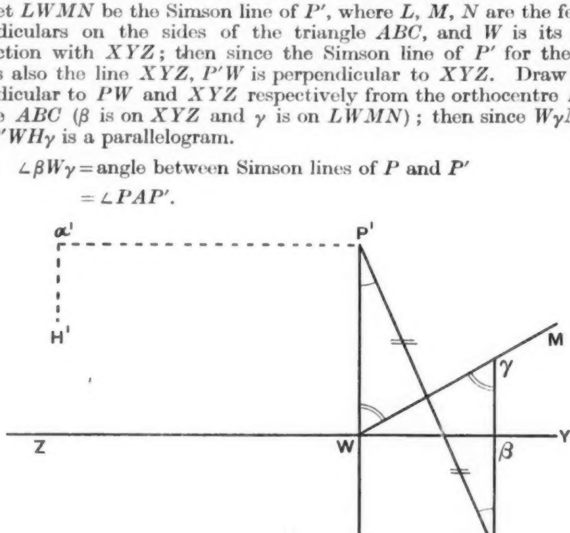
Accordingly, as only one Simson line can be drawn in a given direction, it follows that the point P is the same for all transversals of the triangle ABC which are parallel to XYZ, and so the circumcircles of the orthogonal system of circles $A'B'C'$ all pass through P (the circumcircle dwindles to the point P when the transversal coincides with the Simson line of P).

The Simson lines of P for ABC and $A'B'C'$ are both parallel to XYZ. The Simson lines of P' for these triangles are at right angles.

2. Let $LWMN$ be the Simson line of P', where L, M, N are the feet of the perpendiculars on the sides of the triangle ABC, and W is its point of intersection with XYZ; then since the Simson line of P' for the triangle AYZ is also the line XYZ, $P'W$ is perpendicular to XYZ. Draw $H\alpha$, $H\beta\gamma$ perpendicular to PW and XYZ respectively from the orthocentre H of the triangle ABC (β is on XYZ and γ is on $LWMN$) ; then since $W\gamma M$ bisects $P'H$, $P'WH\gamma$ is a parallelogram.

Also $\angle \beta W\gamma =$ angle between Simson lines of P and P'

$$= \angle PAP'.$$



3. Since ABC and $A'B'C'$ are directly similar and P' is their centre of similitude, the latter figure can be derived from the former by rotation about P' through a right angle and by altering the lengths of the radii vectores in the constant ratio (see broken lines in figure),

$$\lambda = P'A'/P'A = \beta\gamma/W\beta = \beta\gamma/H\alpha ;$$

hence the distance of H', the orthocentre of $A'B'C'$, from XYZ is

$$P'W - \lambda . H\alpha = H\gamma - \beta\gamma = H\beta,$$

and is equal to the distance of the orthocentre H from XYZ. Thus XYZ bisects HH'.

E. P. LEWIS.

REVIEW.

The Astronomical Horizon. By Sir JAMES JEANS. Pp. 23. 2s. 6d. 1945. The Philip Maurice Dencke Lecture, 1944. (Oxford University Press)

In a score of pages Sir James Jeans takes us from our own earth and sun, through our own galactic system, to the horizons of present means of observation in the remote extra-galactic nebulae, and he takes us from the ideas of Aristarchus of Samos to the horizons of present knowledge in the ideas of Eddington and Milne. His unfailing genius for exposition contrives to charm our minds through these vast ranges without trace of strain or flurry. He even finds space for some of his inimitable scale models in terms of particles of tobacco smoke, dinner plates, and a shower of rain on the City of Oxford. The lecture is illustrated by reproductions of astronomical photographs, none the worse, in the case of such masterpieces, for the fact that some are familiar ones.

After surveying the observational data, Jeans briefly recapitulates the theory of the expanding universe. This leads to a discussion of the total number N of protons and electrons in the universe. He sketches Eddington's suggestion which he sums up as that " this N-ness of the external world appears as . . . a contribution of our own minds, not to the universe but to our interpretation of it ", and calls this " a suggestive, fascinating and inspiring vision ", though he finds Eddington's own arguments for its validity wholly unconvincing. He then recalls the simple basic considerations which nevertheless indicate that the several occurrences in nature of numbers of the order of the square root of N must represent something fundamental in a true *description* of nature. Then he alludes to Milne's alternative *description* of the expanding universe, in particular to the use of two time-scales and its bearing upon the constants of physics.

In his last page, Jeans enlarges upon this notion of *description*, as opposed to explanation, as the goal of science. Though couched in language appropriate to such a lecture, it presents in succinct form what the reviewer makes bold to regard as the only legitimate philosophical attitude towards physical theory.

W. H. MCCREA.

BIRMINGHAM UNIVERSITY MATHEMATICAL SOCIETY JUNIOR BRANCH OF THE MATHEMATICAL ASSOCIATION.

Under the chairmanship of Dr. Pedoe, the Society has experienced an encouraging revival; meetings have been held every fortnight throughout the session 1944–5, and a good average attendance has been maintained.

Lectures given to the Society include: " The life and work of Srinivasa Ramanujan ", by Professor Watson; " Congruences ", by Mr. Preece; " Mathematical genetics ", by Mr. Waterhouse; " Feet ", by Dr. Kynch; " Stellar catastrophe ", by Dr. Johnson. Particular mention should be made of the first student lecture to be given for two years, " Calculated and uncalculated music ", by Mr. Coleman.

ENID J. NEWCOMBE,
Hon. Secretary and Treasurer.

FOR SALE.

D. E. Smith : *Rara Arithmetica.*

T. L. Heath : *Apollonius of Perga. Conic Sections.*

Arne Fisher : *Mathematical Theory of Probabilities.* Vol. I. 2nd ed., 1928.

Bibliotheca Chemico-Mathematica. Vols. I, II. Compiled and annotated by H. Zeitlinger and H. C. Sotheran.

Offers to Mr. J. B. BRETHERTON,
508 Manchester Road, Paddington, Nr. Warrington.

Mathematical Gazette : Vols. XV–XIX, bound ; Vols. XXIV, XXV, in parts as issued, with indexes ; Index to Vols. I–XV.

Offers to Miss O. J. LACE, Langdale Hall, Victoria Park, Manchester, 14.

Mathematical Gazette : all copies from Vol. XI, No. 162, January, 1923, to Vol. XXVII, No. 277, December, 1943, with Indexes, ready for binding.

Offers to L. A. W. JONES, Berkhamsted School, Herts.

BOOKS RECEIVED FOR REVIEW.

E. Artin. *Galois Theory.* 2nd edition. Pp. 82. $1.25. 1944. Notre Dame Mathematical Lectures, 2. (Notre Dame, Indiana)

H. Brandenburg. *Sechsstellige trigonometrische Tafel.* Pp. xxiv, 304. $5. 1945; lithoprinted from the edition of 1932. (Edwards, Ann Arbor, Mich. ; Scientific Computing Service, W.C. 2)

F. Emde. *Tables of elementary functions.* Pp. xii, 181. $3.20. 1945 ; lithoprinted from the edition of 1940. (Edwards, Ann Arbor, Mich. ; Scientific Computing Service, W.C. 2)

L. R. Ford. *Nomography* ; **A. H. Copeland.** *Probability* ; **E. Artin.** *Complex Functions.* Pp. 70. $1.25. 1944. Notre Dame Mathematical Lectures, 4. (Notre Dame, Indiana)

Sir James Jeans. *The astronomical horizon.* Pp. 23. 2s. 6d. 1945. (Oxford University Press)

Z. Jordan. *The development of mathematical logic and of logical positivism in Poland between the two wars.* Pp. 47. 2s. 6d. 1945. (Oxford University Press)

K. Menger. *Algebra of Analysis.* Pp. 50. $1. 1944. Notre Dame Mathematical Lectures, 3. (Notre Dame, Indiana)

P. A. Piza. *Fermagoric triangles.* Pp. 153. N.p. 1945. Publication No. 1 of the Polytechnic Institute of Puerto Rico. (Soltero Santurce, P.R.)

W. T. Pratt. *Worked examples in electrotechnology.* Pp. 262. 12s. 6d. 1945. (Hutchinson)

A. Robson. *The earth and the sky. Elementary mathematics.* Pp. 64. 2s. 6d. 1945. (Bell)

A. Wald. *On the principles of statistical inference.* Pp. 47. $1. 1942. Notre Dame Mathematical Lectures, 1. (Notre Dame, Indiana)

S. A. Walling and J. C. Hill. *The velocity-triangle computer in principle and practice.* With xylonite computer. Pp. 36. 7s. 6d. 1945. (Cambridge University Press)

Reports of a Mathematical Colloquium. Second series, Nos. 1–6. Edited by K. Menger. Pp. 64, 48, 55, 55, 79. $1. 1945. (University Press, Notre Dame, Indiana)

PRINTED IN GREAT BRITAIN BY ROBERT MACLEHOSE AND CO. LTD.
THE UNIVERSITY PRESS, GLASGOW

1928.
ed by

on.
V, in

14.
23, to

athe-

1945;
uting

inted
rvice,

tions.
iana)
niver-

oland

atical

f the

1945.

1945.

Dame

ctice.

nger.

Vo

I H
it
do
An
the
log
" t
sar
con
N
use
res
live
by
in c
rea
By
the
wis
glu
All
be
wit
nes
the
I
use
Lag
and

*
194